Chapter 6:
Growth Rates, Multiples, Weighted Average Cost of Capital and the Discounted Cash Flow Model

Introduction

When considering valuation pitfalls and potential errors in valuation of debt and equity, the most important source of an error is making a mistake in developing economic assumptions for revenues. These mistakes discussed in chapter 1 include: (1) assuming that companies can earn a rate of return substantially higher than the cost of capital in competitive industries with relatively easy entry; (2) not recognizing the effects of capacity expansion in an industry relative to demand (which often occur after the market value of capital increases); and, (3) ignoring shifts in the cost structure and demand change in an industry which can quickly render existing assets obsolete. This chapter addresses pitfalls in valuation of corporations that occur for reasons other than making these types of bad assumptions. These problems are related to mechanical implementation of valuation models involving measurement of terminal value, use and interpretation of P/E and EV/EBITDA multiples, computation of weighted average cost of capital consistency between various assets and liabilities included in the bridge between enterprise value and equity value and other practical issues associated with the discounted cash flow model. In virtually any analysis that is derived from an explicit period of earning free cash flow or dividends followed by realizing a lump-sum terminal value, it is the explicit or implicit assumptions used to derive the terminal value assumption that is the largest driver of the valuation. This chapter addresses various financial and mechanical issues associated with various aspects of the discount cash flow model ("DCF model") including measurement of weighted average cost of capital, computation of the bridge between enterprise value and equity value and derivation of terminal value using traditional formulas as well as earnings multiples.

Many ideas about valuation such as the definition of the weighted average cost of capital, free cash flow and net debt are taken for granted by finance professionals, students and academics without working through the underlying valuation logic. However, when some of the most basic valuation concepts are tested with relatively simple mathematical equations and proofs, a few very fundamental ideas that are the building blocks for the DCF model become ambiguous. For example, enterprise value is computed as the present value of free cash flow discounted at the weighted cost of capital which is also the present value of equity plus the present value of net debt. However, when one adds the present value of equity cash flow at the equity discount rate to the present value debt at the debt, the sum does not equal the enterprise value (even when the capital structure is identical on a market basis and taxes are properly considered). When applying the discounted cash flow model, the terminal value is universally discounted at the same rate as the free cash flow over the explicit period, even though the company is assumed to have stable cash flow after the terminal period and should be less risky with less volatile cash flow. The common practice of de-levering and then re-levering the beta in applying the discounted cash flow model is mathematically incorrect if the cost of debt is different from the risk free rates. Other examples like these are discussed below.

Much of this chapter addresses issues related to practical application of valuation using the discounted cash flow model and multiples such as the P/E ratio and the EV/EBITDA ratio. The chapter is not intended to be a typical textbook treatment of discounted cash flow analysis that describes how to compute free cash flow and then add the terminal value and discount the cash flow at the WACC. Instead, the focus is on biases that occur in the DCF model from incorrectly considering stable rates of earnings growth, capital expenditures, deferred taxes and working capital. When addressing valuation multiples, rather than discussing issues like how to select samples for applying the P/E and EV/EBITDA ratios, financial models are developed that demonstrate which factors – asset life, inflation, growth, risk premium, return, taxes and fade periods or transition factors for each variable -- are the most important drivers of the multiples. Concepts of stable ratios of capital expenditures, deferred taxes given tax and book lives are demonstrated in computing multiples when growth rates, returns and cost of capital is

changing over time. Once DCF concepts are integrated with theoretical discussion of multiples, the theoretical drivers of the DCF model and multiples are tested using actual data with regression analysis. The practical application of such regression analysis is to apply real data on estimated growth, return and measurable cost of capital factors to arrive at multiples that can be applied in the DCF model. The final part of the chapter discusses technical issues associated with the treatment of items should be included in the bridge between equity value and enterprise value.

Many of the complexities in computing terminal value and deriving multiples are illustrated on the graph and table below which demonstrates theoretical drivers of the EV/EBITDA ratio. The EV/EBITDA ratio is computed from simulation of free cash flow, use of the growth rate formula and a formula that uses the return, the growth rate and the cost of capital. All of the factors that drive the EV/EBITDA ratio are included in the figure including the life of the assets, tax rates, transition factors, rates of return, costs of capital and growth rates. The graph and table demonstrates that when traditional multipliers using a constant growth model or a simple value driver formula from return on invested capital and growth are applied, the valuation produces mechanical errors.

Book Life (Asset Replacement)	30
Tax Life	10
Tax Rate	30.00%
Working Capital to Plant	13.00%
Transition Years - Growth	6
Transition Years - ROIC	30
Transition Years - WACC	30
Short Term Period	7

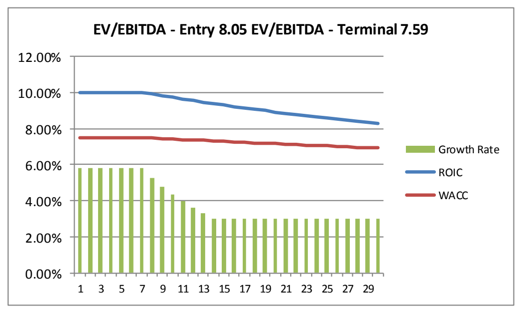
Sensitivity

	Current	Long-term
ROIC	10.00%	7.80%
WACC	7.50%	6.72%
Growth	5.80%	3.00%

LT = ST

Reset

Year of Valuation	1	7
EV/EBITDA - Theoretical	**8.05**	**7.59**
EV/EBITDA - Value Driver	12.63	8.45
EV/EBITDA - Growth Rate	12.63	5.75



Underneath the above seemingly simple analysis are a host of complex modeling and financing issues that are addressed in this chapter. Some of these issues which are discussed in detail include:

- Making reasonable valuations of corporations requires explicit or implicit assumptions with respect to the long-term growth, profitability and risk. For companies that are growing faster than the overall economy and that are earning returns substantially above their cost of capital, one should assume that growth and returns stabilize, but the time frame for stabilization, the transition to stabilization and the long-term profitably are virtually impossible to predict.
- Various different practical formulas derived from stable growth rates can be used to compute terminal value. The stable growth rates should be consistent with other parameters of a valuation analysis including capital expenditures to depreciation, depreciation rates on net plant, working capital changes to EBITDA and deferred taxes as a percent of capital expenditures.
- Deriving implied P/E ratios and implied EV/EBITDA ratios from stable growth rates along with assumptions about the cost of capital after an assumed stable period is reached. These ratios can be used to compute terminal values and can be adjusted for transition periods.
- Working through a plant life cycle to compute the stable ratio of depreciation to capital expenditures, the stable ratio of deferred taxes to capital expenditures and the stable depreciation expense as a percent of net plant given constant growth over the plant life. When the growth rate changes, the capital expenditure and depreciation ratios also change and modeling changes in growth rate require evaluating multiple plant life cycles.
- Combining stable ratios of capital expenditures to depreciation, depreciation rates on net plant and deferred tax to capital expenditures with assumptions about inflation, real growth, return on invested capital and risk premiums to derive enterprise value and the EV/EBITDA ratio for alternative time periods.

Free Cash Flow and Weighted Average Cost of Capital

Before addressing detailed and sometimes complex approaches to compute terminal value, valuation multiples and the bridge between equity value and enterprise value, fundamental issues involving how to compute cash flow and calculate discount rates from are reviewed. Most of these concepts – free cash flow, un-leveraged discount rates, and the CAPM – are not generally disputed in academic circles and are covered more comprehensively in many text books.[1] Some of the valuation basics that are now a standard part of financial analysis -- and are in some cases wrong -- are summarized below.

(1) Valuation of an asset or the value of an enterprise should in theory be measured by discounting free cash flow over its remaining life which is generally assumed to be indefinite in the case of corporations. Theorists insist that using alternative measures such as comparative earnings per share, equity cash flow, payback period analysis, or the accounting return on equity are inappropriate (even though the methods are commonly used in practice). The definition of the value of an enterprise which is equal to the value of the debt, equity and other claims on the company (more complex adjustments to net debt are discussed below).

Enterprise (Asset) Value = Equity Value + Net Debt Value

or, using different terms:

Enterprise Value = Market Capitalization + Market Value of Net Debt

Where,

Market Capitalization = Number of Shares x Share Price

Net Debt = Value of Long-term Debt + Short-term Debt + Minority Interest – Surplus Cash

(2) The value of net debt in the above equation is the discounted present value of future cash flows received by debt holders – interest and principal payments – measured at a discount rate that reflects the possibility of losing money from a default on the debt. The discount rate for the debt cash flows can be computed as a risk free rate of interest associated with the maturity of each debt issue plus the credit spread that reflects the expected probability of default and the loss given default. Net debt is used rather than gross debt because if a company has extra cash that it does not need for operating needs, the cash could theoretically be used to pay down debt. (As discussed below, debt obligations increase the risk and the beta for equity, while cash balances decrease risk and the equity beta unless they are invested in risky securities.)

Debt Value = ∑ Present Value (Debt Cash Flow, Discounted at Risk Free Rate plus Credit Spread)

Valuation of debt illustrates that it is the risk of prospective cash flow that drives value and that this risk measurement has nothing to do with the historic interest rates which determine the level of the debt cash flow. In a very simple case for a zero coupon debt issue providing 100 of cash due in ten years where the credit spread is 2% and the risk free rate is 5%, the current value of the debt is 50.36 as demonstrated below. The discount rate is computed as (1+Rf) x (1+credit spread)-1 and the present value is the cash flow divided by one plus the discount rate raised to the tenth power:

[1] Brealy and Meyers, Principles of Corporate Finance, Chapter 12, Irwin/McGraw Hill, 2000.

Credit Spread	2.00%
Rf	5.00%
Discount Rate	7.10%
Cash Flow	100.00
Present Value	50.36

(3) In an analogous manner to the valuation of debt, the value of equity is the present value of future equity cash flow which can be defined as dividends plus capital gains less new issuances of equity. The discount rate used in computing equity cash flow should be the minimum rate of return required on investments with comparable risk or the cost of equity capital which can also be broken down into a risk free rate and a risk premium. Traditional ways of computing the cost of equity capital are to apply the CAPM or the discounted cash flow model which derives the cost of capital from the long-term expected growth rate in dividends (chapter 4).

Equity Value = ∑ Present Value (Equity Cash Flow, Discounted at Cost of Equity)

Assuming that no dividends are realized on an investment and the stock will be sold in 10 years producing a value of 200 and that the risk premium on the investment is 5.5%, the value of equity is computed in a similar manner as the value of debt illustrated below.

Risk Premium	5.50%
Rf	5.00%
Discount Rate	10.78%
Cash Flow	200.00
Present Value	71.88

(4) Free cash flow is earnings before interest, depreciation and taxes less capital expenditures and working capital changes and it should be computed on an after-tax basis. The total free cash flow is also the sum of the cash flow received by equity investors plus the cash flow received by debt investors minus the tax shield realized from issuing debt;

Free Cash Flow =
EBITDA – WC Change – Capital Expenditures – Operating Tax + Deferred Tax Change

where,

Operating Tax = EBIT x Marginal Tax Rate

or,

Free Cash Flow = Debt Cash Flow + Equity Cash Flow - Interest Tax Shield

In contrast to equity cash flow (dividends and equity issuances), the free cash flow cannot be plucked from items that are shown on the financial statements. When computing free cash flow, debt financing and income taxes complicate measurement because free cash flow is computed on a hypothetical basis as if no debt were present to finance the asset. The presence of debt increases the variation in equity cash flow for a given amount of variation in revenues and expenses. This implies that if the discount rate applied to equity cash flow should change as the amount of debt financing changes.

(5) The reason the interest tax shield is deducted from equity and debt cash flow in the formula above is that benefits from the value of the tax shield are incorporated in the weighted average cost of capital calculation (the debt cost is computed after the tax shield). The investor value is reconciled through applying a lower WACC that increases the value of the cash flow (one can also think of the formula as reflecting the net cash flow paid to debt holders after deducting tax benefits of debt.) Through including the tax shield in the free cash flow formula, the free cash

flow is less than the cash flow received by equity and debt investors. Because the weighted average cost of capital uses incremental debt cost of capital (the risk free rate plus the credit spread), the value of the tax shield reflects the incremental cost of debt rather than the actual interest deductions. In theory, the difference between the interest tax shield generated by actual interest and the interest deductions generated by the incremental cost of debt in the WACC formula should be included in the bridge between enterprise value and equity value.

(6) Because free cash flow consists of debt and equity cash flows, and the enterprise value is defined as the sum of equity and debt value, the value of the overall free cash flow (the sum of equity and debt cash flow) can be used to derive the enterprise value. This calculation works as long as the discount rate applied to equity cash flow is the cost of equity and the debt cash flow is used in valuing the free cash flow;

Enterprise (Asset) Value =
∑ Present Value (Debt Cash Flow, Discounted at Risk Free Rate plus Credit Spread) +
∑ Present Value (Equity Cash Flow including Tax Shield, Discounted at Cost of Equity)

which should also equal:

Enterprise (Asset) Value = ∑ Present Value (Free Cash Flow, discounted at WACC)

In the a very simple case where the terminal value of equity cash flow and debt cash flow occur in one period, the present value of equity plus the present value of debt discounted at their respective discount rates equals the present value of free cash flow discounted at the WACC. The present value of the debt plus equity equal 274.45 and the present value of 300 discounted at the WACC of 9.31% also equates to 274.45. This example is illustrated below using the cost of equity and the cost of debt from the above simple points. Note that when the discounting is for more than one period this equality does not exist.

Period	0	1				
	PV of Future Cash Flow	Future (Term) Cash Flow		Cost of Capital	Market Percent	WACC
Equity Cash Flow (Dividend)	181.00	200.00	Equity	10.5%	65.95%	6.92%
Debt CF (Interest + Repayment)	93.46	100.00	Debt	7.0%	34.05%	2.38%
Total Free Cash Flow (EBITDA - Taxes - Cap X)	274.45	300.00	Total			9.31%
Enterprise Value from PV of Free Cash Flow	274.45	300.00				

(7) In order for the present value of free cash flow to correctly reflect the value of equity plus the value of debt, the WACC must be weighted using market values and the cost of debt and the cost of equity must be incremental costs. When summing the debt and equity values to derive free cash flow, the relative proportion of equity cash flow and debt cash flow that influences drives the total free cash flow and the enterprise value depends on the future values of the equity and debt, not the historic amounts of debt and equity invested. For example, consider a case where the equity value is very high relative to the value of debt because of high expected growth in cash flow while the book value of debt is similar to the value of equity (prospective value relative to book value can be illustrated by the case of Facebook in the film Social Network where the market value had nothing to do with the couple of thousand that was invested to launch the company.) In this example, the value of equity at the market value is much more than the value of debt and the weighting of future free cash flow must account for this fact. This means that weighting percentages used in computing the average cost of debt and equity capital for application to free cash flow must be calculated from market values and not book values. The components of the WACC include:

Debt Discount Rate = Current Risk Free plus Credit Spread from Future Risk

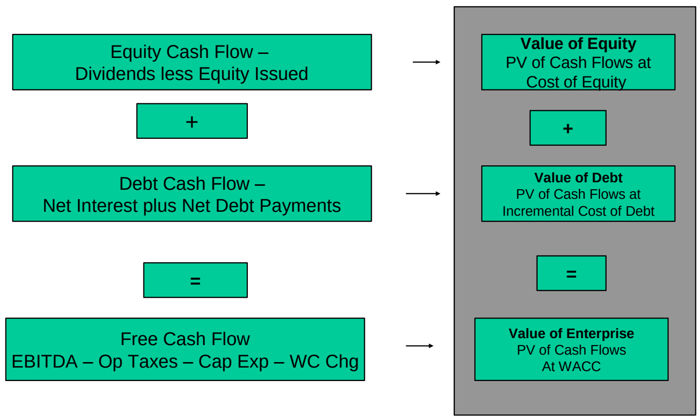
Equity Discount Rate = Required Rate of Return that Reflects Risk of Future Equity Cash Flows

Debt Percent in WACC = Market Value of Net Debt/Enterprise Value

Equity Percent in WACC = Market Value of Equity/Enterprise Value

The diagram below illustrates how since free cash flow is the sum of equity and debt cash flows and since enterprise value is the sum of debt and equity value, that the enterprise value is the present value of free cash flows.

Equity Cash Flow, Debt Cash Flow, Free Cash Flow and Cost of Equity, Cost of Debt and WACC



Since the value of the enterprise is the value of debt plus the value of equity, the value of equity is the enterprise value less the value of the debt.

Equity Value (Equity Cash Flow, discounted at equity Cost) =
Enterprise Value – Net Debt Value (Debt Cash Flow, discounted at debt cost)

(8) The effect of the tax deduction of interest expense on debt is accounted for in the WACC calculation rather than in the cash flow in classic application of the discounted cash flow. The WACC is lower from the WACC which increases the enterprise value. The lower WACC is the reason operating and not actual taxes are used in the free cash flow calculation;

	Percent	Cost	Tax	Weighted
Debt	Market	Incremental	1-Marginal Tax Rate	Mkt Wt x Incremental Cost x (1-Tax Rate)
Equity	Market	Incremental		Mkt Wt x Incremental Cost
WACC				Sum

(9) In the weighted average cost of capital calculation, the equity return increases in proportion to the amount of debt in the capital structure, keeping the overall cost of capital constant. This is often applied through computing un-leveraged betas and then re-leveraging the betas as illustrated in the equations below which assume no tax:

Asset Beta = Equity Beta x Market Equity/Market Capital + Debt Beta x Market Debt/Market Capital

If the debt beta is zero,

Asset Beta = Equity Beta x Market Equity/Market Capital

In terms of the equity beta:
Equity Beta = Asset Beta x Market Capital/Market Equity

(10) The classic weighting in the WACC calculation that is universally taught and used in investment banking analysis does not measure the value of debt and equity when the holding period is more than one period. From a mathematical standpoint, it is not true that the equity discounted at the equity cost of capital plus the debt discounted at the debt cost of capital equals free cash flow discounted at the WACC. This distortion occurs because of the fact that (A+B) raised to the power of more than one does not equal A to the power plus B to the power. In the case of the weighted average cost of capital, one can think of the cash flow of debt as A and the cash flow of equity and B as the cash flow of debt. In terms of a simple example with one future cash flow for debt and one cash flow for equity:

Debt Cash/(1+Debt Cost) + Equity Cash/(1+Equity Cost) < Free Cash/(1+WACC),

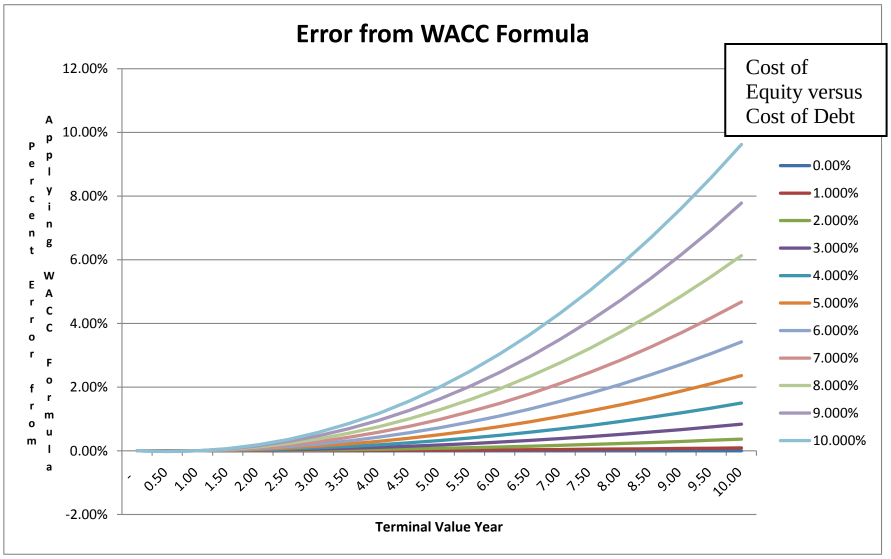
Where:

WACC = (Debt Cash/Free Cash) x Debt Cost + Equity Cash/Free Cash x Equity Cost

To see this, consider a simple model where cash flows – both debt and equity -- are received in a lump sum in ten years. The debt cash flow received in ten years is 100 and the equity cash flow received in ten years is 200. The actual value of debt and equity is 75.9, but the valuation of the combined 300 of cash flow at the computed WACC results in a valuation of of 118.02. The example shows that the incorrect weighting with a large difference between the equity cost and the debt cost causes a large error in the computation of WACC.

Period	0	10				
	PV of Future Cash Flow	Future (Term) Cash Flow		Cost of Capital	Market Percent	WACC
Equity Cash Flow (Dividend)	14.51	200.00	Equity	30%	19.11%	5.73%
Debt CF (Interest + Repayment)	61.39	100.00	Debt	5%	80.89%	4.04%
Total Free Cash Flow (EBITDA - Taxes - Cap X)	75.90	310.00	Total			9.78%
Enterprise Value from PV of Free Cash Flow	118.02	300.00				
Difference: Equity + Debt versus PV of FCF	42.12					
Percent Difference with WACC Problem	55.49%					

In the above example, there is a large difference between the equity and the debt cost and the cash flows are assumed to occur ten years into the future. The value of the equity and the value of the debt is the cash flow discounted at the respective rates. The value computed using the traditional WACC of 118.02 is much higher than the value of the debt and equity of 75.90 because of this simple mathematical issue. The corrected WACC can be computed through dividing total future value of cash flows by the present value of cash flows and compounding over the number of years evaluated.



WACC = (Future Value of Debt + Equity Cash)/(Present Value of Debt + Equity)

The remarkable thing about this relatively minor point is that it is completely ignored in finance texts and in teaching WACC in business schools.

(11) When using a financial projection model, the inflation rate should be consistent with the discount rate and the discount rate should be above the inflation rate reflecting a real interest rate. If the inflation rate is changing, then the discount rate should also change. Further, when discount rates change, the present value factors must be compounded rather than calculated from spot discount rates. For example, consider an extreme example where the inflation rate is 100% in one year and then it declines to zero for the remaining forecast horizon. If there is no cash flow in the year the inflation rate is 100%, using the standard NPV formula with spot rate would not account for the decline in the value of future cash flows from the decline in purchasing power when the inflation rate is 100%. With changing inflation rates or changing discount rates, the discount rate should be computed using the following formulas:

Step 1: Compute compounded index: current index = prior index x (1 + current discount rate)
Step 2: Discount factor = 1/current index

If multiple currencies are being discounted, the same value should be derived whether valuation is made in a common currency or whether the valuation is made in different currencies and then the present value is converted to the common currency. The real interest rate, the company risk premium, the country risk premium and the real terminal growth rate should all be the same and not depend on the currency of the model. The only factor depends on the currency is the inflation rate and if the prospective exchange rate is made to be a function of the inflation rate then the value should be identical whether the model is expressed in the common currency or in the original currencies. If the forward exchange rate is different than the rate implied by differential inflation, then the models should be stated in the original currency.

(12) Just as value can be separate between enterprise value and equity value plus debt value, a series of other ratios and valuation metrics can be viewed from alternative perspectives. For example, the return on equity is an important ratio in judging the performance of management as it measures the money provided to investors relative to the investment they have made. The return on invested capital can be computed in an analogous way as illustrated below. The return on debt is the interest expense plus the interest tax shield divided by the net debt and taxes are

net income multiplied by t/(1-t). Using these two equations it can be demonstrated that the EBIT x (1-t) is the same as NI + Interest x (1-t).

Return on Equity = Net Income/Equity Invested

and,

Return on Debt = Net Interest Expense x (1-t)/Net Debt Issued

then,

Return on Capital = (Net Income + Net Interest x (1-t))/(Equity + Net Debt)

or,

Return on Capital = EBIT x (1-t)/Capital Invested

Given the frequency of using EBIT x (1-t) in this formula and in other circumstances, the number is often labeled NOPLAT or net operating profit less adjusted capital. Reconciliation of the return on invested capital from the return on debt and return on equity is illustrated on the table below. As stated in chapter 3, computing the historic and the projected return on invested capital is an important step in verifying that reasonable assumptions are made in the model.

Tax Rate	40%			
		ROE	13.00%	NI/Equity
EBITDA	1,000			
Less: Depreciation	100	ROD	3.75%	Interest x (1-t)/Debt
EBIT	900			
Less: Net Interest	250	ROIC	7.71%	(Interest x (1-t) + NI)/(Debt + Equity)
EBT	650			
Less: Taxes	260	ROIC	7.71%	(EBIT x (1-t))/(Debt + Equity)
Net Income	390			

	Amount	Percent	Return	Weighted
Equity	3,000	43%	13.00%	5.57%
Debt	4,000	57%	3.75%	2.14%
Return on Invested Capital				7.71%

Stable Ratios and Drivers of the P/E Ratio and the EV/EBITDA Ratio in Discounted Cash Flow Analysis

The discounted cash flow model is the culmination of a series of finance theories including Modigliani and Miller, the Capital Asset Pricing Model and notions that companies eventually reach some kind of stable equilibrium. It is considered the central valuation approach by academics and, in its pure form the valuation approach is not dependent on the opinions of other analysts – which is sometimes called relative valuation -- as is the case when multiples such as the P/E ratio and EV/EBITDA are used. The DCF analysis should derive value from reflecting the fundamental factors that drive the value including return on capital, cost of capital and growth rates. However, the discounted cash flow model, particularly using pure inputs (i.e. without using multiples), is fraught if practical and theoretical difficulties. Problems in computing the cost of capital have already been discussed in chapter 4 relating to the accuracy of beta in measuring risk and difficulties in assessing the equity market risk premium. These cost of capital problems can translate into such wide ranges of valuation when applying the discounted cash flow model that the whole process can become all but useless in practical situations. Further, difficulties in measuring the cost of capital are compounded by required assumptions with respect to long-term stable growth rates in the calculation of terminal value. Applying growth rates in the terminal value formula

without being very careful about adjustments to depreciation expense, working capital changes, capital expenditures and deferred tax can lead to biased valuations.

There is a high dependence in the discounted cash flow model on WACC to compute the present value of cash flows and the assumed growth rate in and/or the multiples applied to projected earnings. It is well known to practitioners that computing the discounted value of free cash flow is subject to a lot of bias and manipulation. All one has to do is ask somebody who has made a valuation analysis using the discounted cash flow technique to see how much the results can be fudged through tinkering a bit with the terminal growth, the discount rate or one of the assumptions. An illustration of the classic DCF problem is shown in extract below from a sell side stock analyst presentation. In the second table, the WACC only changes from 5.7% to 6.3% -- a very small variation given all of the problems and uncertainties associated with the capital asset pricing model. Further, the terminal growth rate varies between 2.5% and 3.1% which again is a very small variation considering all of the unknowns about the future state of the economy, future inflation and other company specific factors. Given this small variation in the cost of capital and the growth rates, the value of the stock varies from a low of 31.96 to a high of 65.43 or a range of more than 100%. This variation in valuation that results from small changes in variables – WACC and growth -- that are extremely difficult to assess renders the model useless even if the model faithfully applies all of the financial theories.

Valuation

We always find the greatest challenge with the ITC story coming from valuation, largely because there is no good comp group for the stock. Accordingly, we focus our efforts predominately on DCF valuation to take into account the large capital spending program over coming years and higher level of free cash generation at the end of the capital investment cycle. Exhibits 4 and 5 look at implied fair values for ITC under different discount rate and terminal value assumptions. Using the two methodologies (terminal multiple and perpetual growth), we are comfortable with a $46 fair value for the stock before taking into account the incremental value drivers identified in Exhibit 1.

Exhibit 4: DCF Valuation: Terminal Multiple

Terminal EBITDA Multiple	Discount Rate						
	5.70%	5.80%	5.90%	6.00%	6.10%	6.20%	6.30%
9.25x	43.82	42.87	41.94	41.02	40.11	39.22	38.33
9.50x	45.64	44.67	43.72	42.78	41.85	40.93	40.03
9.75x	47.46	46.47	45.49	44.53	43.58	42.64	41.72
10.00x	49.28	48.27	47.27	46.28	45.31	44.35	43.41
10.25x	51.10	50.07	49.05	48.04	47.05	46.07	45.10
10.50x	52.92	51.86	50.82	49.79	48.78	47.78	46.79
10.75x	54.74	53.66	52.60	51.55	50.51	49.49	48.48

Source: Company data, Credit Suisse estimates

Exhibit 5: DCF Valuation: Perpetual Growth

Terminal Growth Rate	Discount Rate						
	5.70%	5.80%	5.90%	6.00%	6.10%	6.20%	6.30%
2.50%	48.33	45.18	42.22	39.43	36.80	34.33	31.98
2.60%	50.72	47.40	44.28	41.36	38.60	36.01	33.56
2.70%	53.27	49.76	46.48	43.40	40.51	37.79	35.22
2.80%	55.99	52.28	48.81	45.57	42.53	39.67	36.98
2.90%	58.91	54.98	51.31	47.88	44.67	41.67	38.84
3.00%	62.05	57.86	53.97	50.35	46.96	43.79	40.82
3.10%	65.43	60.97	56.83	52.98	49.40	46.05	42.92

Source: Company data, Credit Suisse estimates

The reason for such wide differences in the above table is entirely due to the manner in which terminal value is computed using the formula:

$$\text{Terminal Value} = (\text{Last Period Cash Flow} \times (1+g)/(\text{WACC}-g))/(1+\text{WACC})^{\text{number of periods}}$$

This formula uses WACC in two places, both in computing the terminal value in the last period in discounting the value to the present. To understand problems and alternatives when applying the DCF model, alternative approaches to the terminal value can be compared. To consider whether a reasonable practical alternative can be developed along with a method that yields numbers that can be applied, four different methods for computing the terminal value are discussed below:

- Application of a stable growth formula above that uses the last or stable cash flow estimate along with a constant growth rate assuming the cash flow grows in perpetuity. Advantages of the constant growth method are that it does not depend on the valuation made by others and it can be applied with a simple formula. Disadvantages are that it does not account for changes in cash

flow that arise from changes in rate of return and that it results in a very wide range of results as illustrated above.

- Employment of a formula that considers both the growth rate and the prospective return that can be applied to income. This formula, labeled the value driver formula below and defined as (1-growth/return)/(cost of capital – growth), accounts for the possible convergence in returns to cost of capital over the long-term. One disadvantage of this formula is that it is unclear what implicit assumptions are made with respect to transition periods from the current earnings to return on capital and cost of capital convergence.

- Use of the EV/EBITDA or another multiple in deriving the terminal value. Advantages of this method are the range in values are less than the growth rate formula and that values are consistent with market estimates (in the above table taken from an actual presentation, the range in growth value is between 54 and 38 rather than between 65 and 32). Disadvantages are that the EV/EBITDA multiples from comparative analysis are subject to manipulation of samples and that the EV/EBITDA multiples are not adjusted for differences in the company value as the growth rate of the company slows and as its return may converge to its cost of capital.

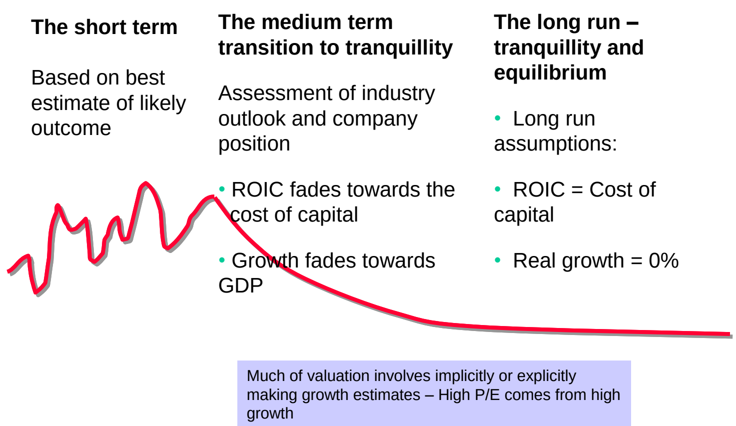
- Development of a formula for the EV/EBITDA ratio that can be applied in the terminal value calculation that explicitly accounts for varying transition growth and return periods, stable ratios of capital expenditures to capital expenditures, stable depreciation rates and stable levels of deferred taxes relative to net plant. This approach forces one to directly consider a host of variables that drive the long-term value including different rates of convergence between return and cost of capital, declines in cost of capital during stable growth periods, as well as changing growth rates and asset replacement. Advantages of this approach are that it is flexible and it reduces large variation in valuations. Disadvantages of the method are that it requires a number of somewhat complicated calculations and it is not commonly used in valuation.

The remainder of this section considers theoretical issues associated with measuring stable growth rates and resolving DCF problems using the four alternative approaches summarized above. The first part discusses theoretical and practical issues associated in applying the stable growth rate formula. The second section turns to the value driver formula and describes general issues related to the convergence of cost of capital and return on capital. This section covers how growth and cost of capital can be translated into a P/E ratio and how the derived P/E ratio can consider changes in real growth, movements in rates of return and variation in the cost of capital. The final section works through calculation of the EV/EBITDA ratio that was illustrated in the table at the beginning of the chapter. This analysis requires development of stable ratios for stable depreciation rates on net plant, stable ratios of capital expenditures to net plant and stable ratios of deferred tax to net plant which depend on the lifetime of plant and the future growth rate.

Stable Growth Rate

Given that corporations are supposed to last indefinitely in the discounted cash flow model (which is a debatable point), some kind of assumption must be made with respect to long-term growth rates, returns on capital and cost of capital after the explicit period of cash flow projections. The general approach in applying the DCF model is that after a relatively short period of about five years, companies will gradually or suddenly begin to stabilize and grow at moderate rates. While it is arrogant to suggest that one can somehow predict that when a company will become stable and stop growing, it is also unreasonable to assume that a company can continue to grow faster than the overall economy for an indefinite period of time. As companies grow quickly, they obviously become very big and achieving a fast rate of growth becomes more difficult. If high growth assumptions are made over long periods, companies eventually become so large that they grow bigger than the whole economy (a similar argument can be made with respect to growth rate of countries – China cannot grow at real rates of 8-10% for an indefinite period without leaving no economic activity whatsoever in the rest of the world). Economic growth is limited by the combination of population growth with productivity growth and it is very difficult to achieve high rates of productivity growth over long periods.

When applying the discounted cash flow model, some assumption must be made with respect to the long-term and it is unreasonable to assume growth rates can occur for long periods above the overall nominal growth rate in the economy. It is also not realistic to assume that every company will eventually simply fail and fall into bankruptcy. As products of a company reach the end of their life cycle or become obsolete, management develops new products and business lines in attempting to sustain high growth and profitability. Given the two extremes -- maintaining growth above the overall economy or simply dying -- an assumption of a stable growth rate has become typical in making valuations. An illustration of the type of growth rate assumptions made in classic DCF analyses is shown in the graph below.[2] The date at which the transition from short-term to long-term growth begins and the length of the transition period is arbitrary.



The great telecom meltdown was discussed in chapter 1 which involved an assumption that very fast growth could be achieved over long periods of time. The fallacy of this assumption is illustrated in the table below which shows the time that it takes for an industry to overtake the economy using different growth rates and different shares of the economy. The table can be easily constructed by simply growing the economy with productivity growth and population growth while growing a particular industry at a different rate. The MATCH function is used to count the columns until the economy becomes the whole economy. The table demonstrates that if a high growth rate is used it is very unrealistic to assume that an industry can grow at high rates. For example, if the growth rate is 20% and the industry is 10% of the economy, it takes only 15 years for the nothing to be left in the economy other than the industry in question. Making assumptions that a company can achieve high growth for long periods of time must be ruled out in valuations.

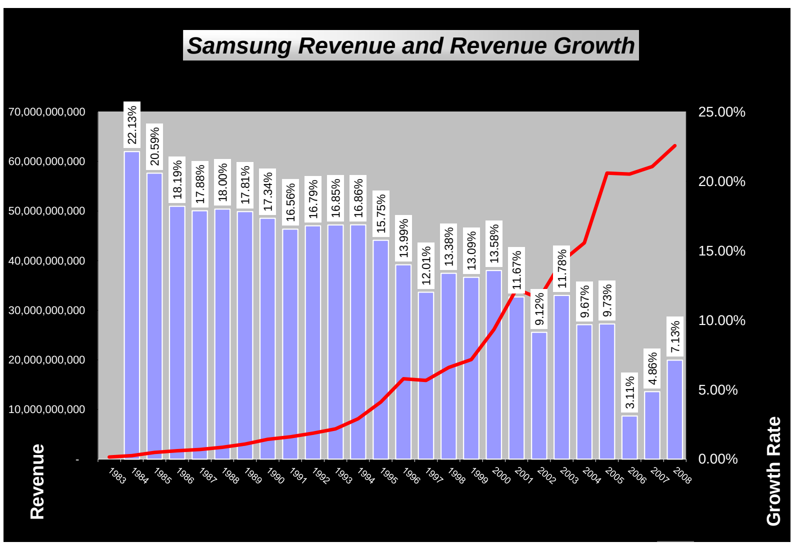
Years to Overtake the Entire Economy

		Initial Percent of Economy			
		2.5%	5.0%	10.0%	20.0%
Industry Growth	5%	101	101	97	68
	10%	53	43	33	23
	20%	24	20	15	11
	30%	16	13	10	7
	40%	12	10	8	6

[2] Growth rate in the transition period can be constructed from the equation: $g_t = g_{t-1}$ x[1/(short-term growth/long-term growth)]^(1/transition period).

To resolve problems with making high growth rate assumptions, investment bankers who perform discounted cash flow analysis generally make a pessimistic assumption that growth in cash flow once a terminal period occurs will be limited to the projected rate of inflation. The typical assumption is that companies will stabilize to a tranquil zero real growth rate in a period of somewhere between five and ten years, perhaps after a smooth transition period until the supposed tranquility is obtained. While the assumption is commonly made, it is difficult to come up with any company -- or person for that matter -- that has reached this kind of tranquil nirvana or has managed such a transition to equilibrium. Further, the valuation analysts do not seem to be concerned about the basic point that if all companies somehow reach this kind of equilibrium where there is no real growth in cash flow, no companies would contribute to real economic growth and the world economy would stagnate in a never-ending recession.

When inspecting individual companies over long time periods, one can find downward trends in growth, but not the sudden change to stable growth rates that is often assumed in the discounted cash flow model. For example as shown below, the growth rate of Samsung Corporation has declined, but it did not happen in five years and it has not come all the way down to the overall rate of inflation (bankers in South Korea are good at retaining a lot of historic data.) Similar examples can be found with many companies that have become large and successful. When reviewing large companies such as Samsung one should keep in mind that there is survivorship bias as smaller companies which have never grown or have failed are more difficult to acquire data. You can get into endless philosophical and economic arguments about growth rates, survivorship bias and reaching a period of stability. But when applying the DCF model, some explicit or implicit assumption is required and assuming a high rate of growth over an indefinite period is not reasonable.



As already discussed, the standard terminal value formula in deriving the terminal value begins with the final year cash flow in an explicit period and applies the formula:

Terminal Value = Final Year Free Cash Flow x (1+stable growth)/(WACC-stable growth)

This formula is derived from the basic idea that a cash flow in perpetuity can be computed as:

Value = Cash Flow$_1$ / Cost of Capital

Since the growth rate applied to cash flow is the reverse of discounting and the cash flow in the next period is the current period multiplied by one plus the growth rate, the formula is the same as assuming the cash flow continually grows and the risk of the cash flow does not change. Because the WACC is in

the denominator of the formula, the terminal value changes by a wide margin when the WACC varies. The formula is also very sensitive to the growth rate as a high growth rate can make the denominator very small and imply a high value. Finally, the growth formula can be easily be translated into the EV/EBITDA ratio as the terminal value is the enterprise value and the free cash flow is a function of the EBITDA.

TV = FCF/(WACC-g) = (EBITDA – Cap Expenditure – WC Change – Tax)/(WACC-g)

While questionable logic (such as assuming world economic growth will stop) and errors in the prediction of growth are pervasive in valuation, the errors in valuation analysis discussed here are not related to under-estimating or over-estimating growth. Precisely because the exercise of predicting growth is so difficult, it would be presumptuous to assert that valuation analyses were flawed when someone made an optimistic or pessimistic estimate. However, it is worthwhile to consider the relationship between growth and cost of capital. Growth rate uncertainty is a big part of the reason that cash flows themselves are uncertain and companies where valuations depend on high growth should have high cost of capital no matter what the CAPM suggests. Given the inherent uncertainty of guessing at what date growth rates will change, investments that do not depend on achieving high growth for a long time period should be valued more highly than investment strategies which cannot easily adjust to changes in growth.

Convergence of Return on Capital and Cost of Capital

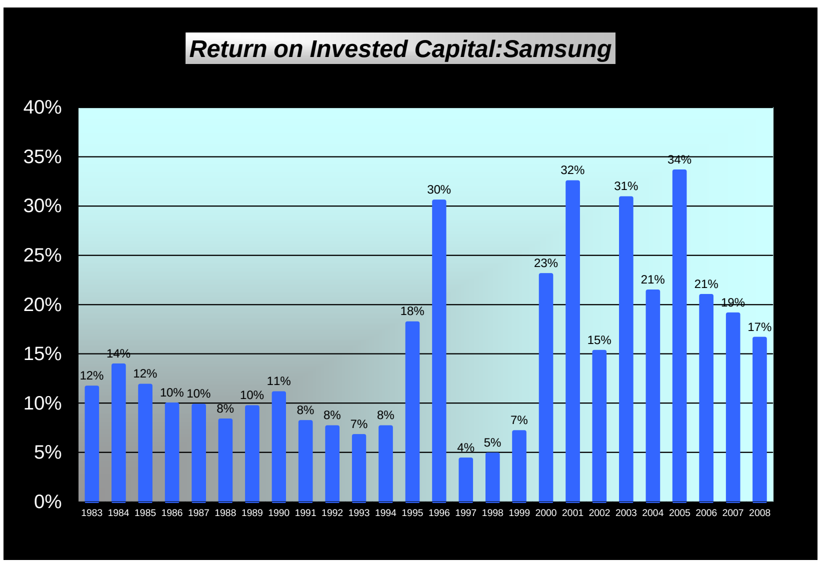
The discussion of growth made it seem as if a company can grow, then the value will increase. While the growth formula does apply two key factors that drive value – the growth rate and the cost of capital – it does not account for the third factor which is the rate of return on capital. The rate of return on capital can change the way growth rate affects value. If a company is earning below its cost of capital, or if it is just earning its cost of capital, then an increase in the growth rate does not increase value. The implication that the growth rate only matters if the return is above the cost of capital means that the return relative to the cost of capital should explicitly or implicitly be a consideration in computing terminal value. Consider a very simple example where the initial investment in a company is 100 and the return is 10% while the cost of capital is 15% -- i.e. the return is below the cost of capital. If the company only lasts for one period and if income is re-invested, then the cash flow realized is 110, but the value declines because the present value factor is 1.15. In this case where the return is less than the cost of capital, the value of the company declines to 95: (100 x 1.10/1.15). This simple example is meant to demonstrate that return on capital should be an explicit consideration in deriving value.

To resolve problems with wide variation in terminal value driven by growth rates, one can attempt to take the growth rate out of the process by assuming that the rate of return equals the cost of capital. If the return is equal to the cost of capital, then the value does not change whether the growth rate is 10% or -5%. All one has to do is to make a seemingly sensible assumption that over the long-run, competition will push returns down to the cost of capital as products become more commoditized. Think about Nokia, Research in Motion (blackberry) and Apple. A few years ago Nokia was rave of investors and the company was assumed to have unique products that would yield a sustainable competitive advantage and strong returns over an indefinite period. Then Nokia lost its luster and Research in Motion was the poster child for investors. A few short years later, RIM lost its popularity and Apple became the most valuable company in the world. While one can make general arguments that the return driven by competitive advantage will converge to the cost of capital, the idea that rate of return will decline to the cost of capital is more difficult to accept than the idea that companies will stop growing. If a company cannot earn more than its cost of capital, it should not be in business and it should not make new investments. Recall the discussion in chapter 4 where it was demonstrated that the market to book ratio is equal to one when the return equals the cost of capital. The fact that the market to book ratio of companies is substantially above one is evidence that the companies do realize a rate of return above their cost of capital.

From an economic perspective, one can make the argument that competition will drive returns to the cost of capital in competitive and mature industries where there is little differentiation in products. However even in these circumstances it makes little sense to assume that company will make new investments without expecting some kind of economic profit. On the other hand, if a company is currently earning

returns far above its cost of capital it is not reasonable to assume that such profit can last indefinitely. Companies can copy products, management techniques, marketing strategies and cost structures which allow some companies to realize extraordinary returns. As with the growth rate, some kind of explicit or implicit assumption must be made.

When inspecting mature companies, it is more difficult to find a consistent trend where the return on capital converges to the cost of capital than to find evidence of declining growth rates. (It is also tricky to compute the return on capital after the company experiences a large write-off for goodwill or asset impairment as the subsequent investment balance declines and the balance sheet no longer represents invested capital.) The example of Samsung Corporation shown below illustrates that return on invested capital has increased rather than declined, refuting the idea of cost of capital converging to return on capital. Many other mature large companies can be observed and it is quite rare to find companies that are just earning their cost of capital and have a return on capital similar to the cost of capital. Some argue that finding evidence of declining returns is challenging because of survivorship bias in the statistical analysis, meaning that companies where returns fall go bankrupt and stop producing things and one cannot find long-term trends for these companies. Others argue that when companies become big, they are very careful to only make investments where the return is above the cost of capital.



Despite difficulties in making the assumption that returns will suddenly converge to the cost of capital, a few points about returns should be kept in mind. Three issues to consider include:

1. When a company is earning a very high return on invested capital, it is reasonable to assume the return will begin converging to the cost of capital although it is not at all reasonable from an economic philosophical perspective that the return will completely converge to the cost of capital.
2. When constructing a financial model, one should be very suspicious of projected returns that are substantially above the historic returns. This implies that computing returns on equity and returns on invested capital after the financial statements are developed should be a standard part of the modeling process.
3. When evaluating returns it is often better to use the return on invested capital rather than the return on equity because the return on equity can be distorted by changes in historic or future capital structure.

One can account for both growth and return on capital relative to cost of capital through applying a formula to the NOPLAT as shown below (recall that NOPLAT is EBIT x (1-t):

$$\text{Enterprise Value} = \text{NOPLAT} \times (1 - \text{asset growth}/\text{ROIC})/(\text{WACC} - \text{asset growth})$$

Through applying growth and cost of capital estimates, the enterprise value can be derived from the differential between the return on capital and the cost of capital as well as the growth rate. If the cost of capital is currently below the return on capital and the return on capital is the incremental cost of capital, then some argue that the formula results in assuming that the existing assets earn the current return while new investments earn an incremental return. Continuing with the approach, if the return on new investments is assumed to be the same as the cost of capital, then the formula implicitly removes the effect of growth from the terminal value and the value of the company is more dependent on the ability of exiting assets to earn an economic profit.

This formula is sometimes labeled the value driver formula as it is driven by three variables – asset growth, ROIC and WACC – rather than only by one variable in the formula above. When applying this formula, the current level of return implied in the NOPLAT may have a different value than the ROIC in the formula. Therefore, it is suggested that return in the formula applies to new investments while the company continues to earn its current return on exiting assets. Since the NOPLAT can be expressed as the current return multiplied by the invested capital, the formula can be re-stated as:

$$\text{Enterprise Value} = \text{Invested Capital} \times ROIC_1 \times (1 - \text{asset growth}/ROIC_2)/(\text{WACC} - \text{asset growth})$$

where the $ROIC_1$ is the current return and the $ROIC_2$ is the future return. When applying this formula, if a company is currently earning a higher return on capital then the future return, the enterprise value will be higher. Similarly, if one company is expected to earn a lower return than another company, its $ROIC_2$ will be lower and it will have a lower value even if the current return is the same. While the formula seems very sophisticated and useful for resolving problems with terminal value, a question arises as to what is the implicit assumption with respect to movement in the return on invested capital over time. Other questions involve what kind of assumptions with respect asset growth rates are required in order to generate implied growth rates in asset value.

The formula can be used to demonstrate that when the return equals the cost of capital, the growth rate does not matter in establishing the enterprise value. Here, the formula boils down to the simple perpetuity formula and the value of the company is simply the current level of invested capital multiplied by the ROIC divided by the cost of capital. To work though this formula, assume that ROIC = WACC, then:

$$\text{Enterprise Value} = \text{Invested Capital} \times ROIC_1 \times (1 - g/\text{WACC})/(\text{WACC} - g)$$

$$\text{Enterprise Value} = \text{Invested Capital} \times ROIC_1 \times ((\text{WACC} - g)/\text{WACC})/(\text{WACC} - g)$$

$$\text{Enterprise Value} = \text{Invested Capital} \times ROIC_1 \times ((\text{WACC} - g)/\text{WACC}) \times 1/(\text{WACC} - g)$$

$$\text{Enterprise Value} = \text{Invested Capital} \times ROIC_1 \times 1/\text{WACC}$$

This formula demonstrates if the return on capital equals the invested capital on future assets, then one can simply use the current after tax profit and assume that any new investments do not generate any value. All of the value of the company comes from the existing assets and the rate of return earned above the cost of capital for those existing assets. While the formula is more flexible than the simple growth formula, it includes a number of simplifying assumptions. Some of these assumptions include:

- The current level of investment is assumed to occur indefinitely and assets are assumed to remain in place without requiring re-investment. If a business is becoming more competitive with increased competition from around the world, there is no particular reason to assume that a very high return on existing assets can be maintained.

- If the return on assets in an industry is declining, it is logical that the return on the re-investment required for the existing assets to also fall implying that the existing asset base should not be used.

- If the assumption of a return equal to cost of capital is used, then the return on new assets is assumed to immediately fall to the cost of capital without any transition period.

- The cost of capital on existing assets is assumed to remain constant, even though there is no prospective risk from making new investments that are intended to generate economic profit.

- The implicit assumption about future return on capital which is a mixture of the return on the existing investments including re-investment in the investments and the new investments is difficult to interpret.

- If companies realize a P/E ratio of 15 times which is the long-term average for the overall market and if the return does not exceed the cost of capital, then the P/E ratio is one divided by the cost of equity capital which implies a total cost of equity of 6.7%.

To see how the value driver formula works, one can begin with establishing the formula for the P/E ratio using ROE, cost of equity, growth in earnings and the level of earnings. Once the formula for the P/E is established, an analogous formula can be developed for the EV/EBITDA multiple using the ROIC and the WACC. While the formula for enterprise value is more useful in computing terminal value, the formula for equity value is more convenient in explaining how an equation can be derived for multiples such as the P/E ratio or the EV/EBITDA ratio. Therefore, in working through derivation of the value driver formula and how to analyze some of the above issues, it is easier to use net income than NOPLAT.

To understand the value driver formula where value is driven by rate of return, growth and cost of capital, begin with the dividend discount model where the price of a stock price is determined by the cost of capital along with an estimate of the growth rate in dividends. By assuming that marginal investors, who are the ones buying and selling shares, believe the growth rate in dividends is constant forever, one can establish the well-known dividend discount equation for computing the value of a share. The value of a share is the next anticipated dividend divided by the difference in the cost of equity and the growth rate in dividends (the mathematics of the formula requires using the next year cash flow rather than the current period dividend):

$$P_0 = D_1/(k-g)$$

In estimating the growth rate in dividends, one can use the sustainable growth formula where the level of future investment depends on re-investment of earnings and the dividend payout ratio. If all earnings are re-invested implying a zero dividend payout ratio, then the equity grows by the net income which is in turn defined by the return on equity. On the other hand, when the dividend payout ratio is 100%, all of the income is re-invested and the growth rate is zero. The growth rate is a function of the return on equity and the payout ratio as reflected in the sustainable growth rate formula:

$$\text{Sustainable growth} = \text{ROE} \times (1\text{-dividend payout})$$

This formula can be re-arranged to be more useful and reflect the dividend payout ratio as a function of growth as follows:

$$\text{Dividend Payout} = 1 - \text{Sustainable growth/ROE}$$

Since dividends equal the dividend payout ratio multiplied by the dividend per share, the dividends can be expressed as:

$$D_1 = (1-g/ROE) \times EPS_1$$

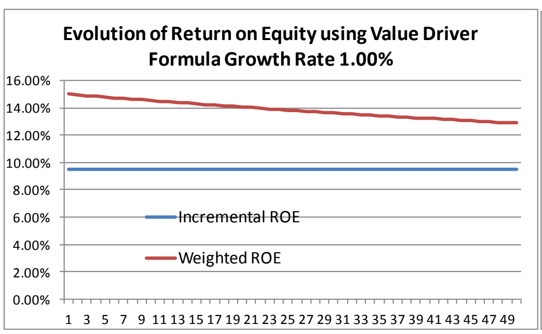
Using this formula, the value and the P/E ratio can be expressed in terms key value drivers – the ability to earn more than the cost of capital and the ability to grow that difference. By substituting the above formula for D_1 into the formula for the current price, the price of the share becomes:

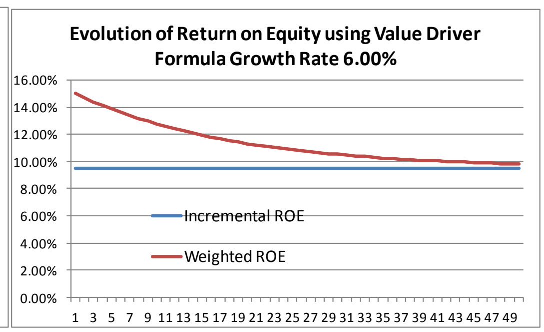
$$P_0 = [1\text{-}g/ROE] \times EPS_1/[k\text{-}g]$$

If the earnings are divided by the price of the share, then the forward P/E ratio can also be defined through simply dividing the price by the earnings in the above equation:

$$P_0/EPS_1 = [1\text{-}g/ROE]/[k\text{-}g]$$

The implications of the value formula can be illustrated by computing a weighted average return on equity that is derived from the current book value of equity and the incremental equity that is generated from growth. A progression of return on equity can be computed through converting the growth to the dividend payout ratio using the formula 1-g/ROE. With the dividend payout ratio and the weighted average ROE, the earnings and the dividends can be established. As the equity capital grows, the proportion of ROE that comes from the initial ROE declines and the proportion that comes from the return on new investment increases. The higher the growth rate, the faster more the weighting is skewed to the incremental ROE and the less to the original ROE. The manner in which the ROE gradually converges to the incremental ROE with different growth rates is illustrated in the two graphs below. In the first case with 1% growth, the ROE remains more than 200 basis points above the incremental ROE after 50 years. With 6% growth, the weighted return converges much faster as more of the equity consists of the new equity. In year 15, the difference in ROE is more than four percent if the growth is 1% while the difference is about 2% with the 6% growth rate.





The graphs above demonstrate some of the problems in applying the value driver formula. If the growth rate is lower, the competitive advantage may be disappearing faster and perhaps the convergence should be faster instead of slower. In either case, the time it takes to converge is relatively long and there is no flexibility to adjust the time frame for moving the convergence of the rate of return. There is no economic theory that suggests returns on existing investments can be maintained (without some kind of contract) and there is no economic theory that implies that low growth companies can maintain returns for longer periods of time. The graphs demonstrate that the reason for lower variability in returns is Finally, the length of time for the returns to converge do not account for depreciation and aging of existing assets.

Using the P/E Ratio in Terminal Value Calculations

If you spend time watching financial analysts on television sounding highly intelligent in their opinions about the value of a stock, they generally pontificate about both the projected earnings and the valuation. The analysts generally make some kind of earnings projection (that is often optimistic) and then apply a future P/E ratio to the projected earnings to arrive at what they think the future price will be. What they have done is compute equity cash flow rather than free cash flow where the future earnings multiplied by

the P/E is the terminal value and the dividends received during the intermediate period are like the explicit free cash flow. These valuations are analogous to the DCF being discussed except the valuation is made with equity cash flow rather than free cash flow. When computing the future price using a future P/E ratio, the financial analysts often seem to have some kind of magic way to project the future P/E ratio that drives the valuation. This section addresses techniques to evaluate the P/E ratio for purposes of computing terminal value (and take the magic out of process). Evaluating the P/E ratio is a precursor to evaluating the EV/EBITDA ratio for use in the DCF that is driven operating characteristics.

The close relationship between the P/E ratio and the EV/EBITDA ratio can be illustrated considering an extreme case for a company with no taxes, no debt and no depreciation. In this case the equity value is the same as the enterprise value and the EBITDA is the same as the net income (EBITDA minus depreciation minus interest minus taxes equals net income). Since the net income equals EBITDA and enterprise value equals the equity value in this very special case, the P/E ratio equals the EV/EBITDA ratio. This means that when factors such as growth, return and cost of capital influence the P/E ratio, these same ideas will affect the EV/EBITDA ratio that is applied in the DCF model. When using a multiple for computing the terminal value, the range in valuations declines dramatically as illustrated in the two tables below. The first table (using the DATA TABLE function in excel) illustrates the wide range in value that arises from variation in the WACC and terminal growth rates. The second table demonstrates the much smaller valuation that occurs when the EV/EBITDA ratio is used instead of the growth rate.

While use of a multiple seems to resolve the problem major problems that exist with the growth rate formula. Use of a multiple such as the EV/EBITDA ratio means that the valuation depends on the opinions of other people. To demonstrate the problem with applying multiples, recall the discussion of valuing homes before the financial crisis of 2008. Appraisers would play games with comparative samples where houses that sold for relatively low value would be excluded from the sample while homes with a high selling price – perhaps because of better features such as location – would be included. As the appraiser arrived at a higher value, the loan would be more and the selling price would be more. The higher selling price of the home in question would then be used in the next appraisal and a viscous circle would be created. To illustrate similar problems in making valuations using multiples, the excerpt below demonstrates how samples in deriving multiples can be very subjective and generally lack logic. The case involves a small freight airline company with a market capitalization of USD 175 million to that was compared with Federal Express with a market capitalization then of USD 5.971 billion.

Table 3
Valuation Table

Air Freight Company Comparables

Share prices as of close: 5/30/97

		52 week:			YTD	EPS		
Ticker	Price	High	Low	Mkt. Cap.	Perf.	FY96A	FY97E	FY98E
ATLS	$ 28.75	$ 59.75	$ 19.88	$ 645.4	-39.8%	$ 1.88	$ 2.10	$ 2.50
KTTY	16.75	17.25	8.00	175.1	67.5%	0.98	$ 1.15	$ 1.45
FDX	52.38	57.88	36.25	5,970.8	17.7%	3.32	4.24	-
ABF	38.25	38.38	19.50	803.3	63.6%	1.28	3.40	4.00

		P/E			P/EBITDA			Ent.Value/EBITDA			P/E vs. SP500		
Ticker	P/B	FY96A	FY97E	FY98E	FY96A	FY97E	FY98E	FY96A	FY97E	FY98E	FY96A	FY97E	FY98E
ATLS	3.0x	15.3x	13.7x	11.5x	1.5x	1.1x	0.9x	8.9x	6.4x	5.0x	0.74x	0.72x	0.64x
KTTY	2.9x	17.1x	14.6x	11.5x	9.5x	5.2x	4.2x	10.3x	5.7x	4.6x	0.83x	0.76x	0.64x
FDX	2.1x	15.8x	12.4x	NA	4.4x	4.0x	3.6x	5.5x	5.0x	4.4x	0.77x	0.65x	NA
ABF	1.9x	29.9x	11.3x	9.5x	3.3x	2.6x	2.4x	4.8x	3.8x	3.5x	1.45x	0.59x	0.53x
Mean	2.5x	19.5x	13.0x	10.9x	4.7x	3.2x	2.8x	7.4x	5.2x	4.4x	0.9x	0.7x	0.6x
Adj. Mean	1.3x	16.4x	13.0x	5.8x	3.9x	3.3x	3.0x	7.2x	5.3x	4.5x	0.8x	0.7x	0.3x
High	3.0x	29.9x	14.6x	11.5x	9.5x	5.2x	4.2x	10.3x	6.4x	5.0x	1.5x	0.8x	0.6x
Low	1.9x	15.3x	11.3x	9.3x	1.5x	1.1x	0.9x	4.8x	3.8x	3.5x	0.7x	0.6x	0.5x

Note:
Enterprise Value = Market Value + LT Debt - Cash and Equivalents
ATLS = Atlas Air
KTTY = Kitty Hawk
FDX = Federal Express
ABF = Airborne Freight

Source: Company reports and Scott & Stringfellow estimates

Problems with use of multiples such as the EV/EBITDA ratio do not arise only because of the relative valuation problem. Another issue involves the fact that multiples such as EV/EBITDA and P/E ratio should change over time as a company matures and grows more slowly and as its return converges to the cost of capital. As demonstrated above, when the return on investment equals the cost of capital, the multiple is simply one divided by the cost of capital, no matter what is the growth rate. However when the return is above the cost of capital and the company is growing quickly, the multiple can be very high. Academics who complain about using multiples in the DCF model insist that that the multiple that currently exists when making comparisons of current companies cannot be used in terminal value calculations. To address problems with relative valuation in assessing multiples, the P/E ratio is with different long-term and short-term growth rates; different short-term and long term returns and changing cost of capital. Through explicitly considering the these factors along with transition periods, the multiples can be derived without resorting to relative valuations and without distorting the valuations for changing growth rates, returns and costs of capital. Through using the implied multiples in the terminal value calculation and computing those multiples value drivers and transition factors, one can enter things like the long-term real growth, the difference between the ROIC and the WACC and the risk premium on the WACC in the DCF analysis.

In modeling the implied P/E ratio from value drivers, a couple of formulas and concepts can make the process much easier. The first idea is to compute transition factors for each variable that can be different in the long-term versus the short-term. If variables have time differentiation (i.e. short-term and long-term values), it is generally not reasonable to assume that the change in the variable suddenly changes in one year. Instead, the variable should gradually change from the short-term rate to the long-term rate. The period over which the variables gradually change is called the transition period. This means the transition period as well as the short-term period should be defined (the long term period is not necessary because it is the sum of the short-term period and the transition period.) In computing values over the transition period, the following two formulas can be used to interpolate assuming a long-term and short-term ROE:

$$ROE_t = (1 + g) \times ROE_{t-1}$$

where,

$$g = (ROE_{long\ term}/ROE_{short\text{-}term})^{(1/(1+transition\ period))}$$

or,

$$ROE_t = ROE_{t-1} + \text{Linear Factor}$$

where,

$$\text{Linear Factor} = (ROE_{long\text{-}term} - ROE_{short\text{-}term})/(1+\text{transition period})$$

To see why the term (1+transiton period) should be used rather than the transition period, consider the case of an immediate transition without a transition period. In this situation, the transition period is zero and the growth rate is $ROE_{long\ term}/ROE_{short\text{-}term}$ while the linear factor is zero. If one was not added to the transition period as in the above the calculations would not be possible in the case of an immediate transition. Considerations about whether to use the growth rate formula or the linear formula depend on whether it is possible to have a negative ROE (or other factor that will be subject to transition and interpolation). If there is a negative number, the more elegant growth rate formula does not work while the less fancy linear factor still applies.

Discounting cash flow can also be used to understand the theoretical drivers of the P/E ratio and the EV/EBITDA ratio. These ratios can be computed by the fundamental drivers of value discussed at the beginning of the book; namely earning a return above the cost of capital and growing the firm. Deriving the P/E ratio and the EV/EBITDA ratio may not be of much value in everyday valuations. However understanding what really drives the ratios can be instructive in thinking about valuations. To understand

the drivers of value and the level of the two ratios, the growth rates can be separated into short-term and long-term growth rates. Further, the cost of capital can be broken down to building blocks – the real rate of interest, the inflation rate and the risk premium. Each of these factors can be differentiated by time period as with the growth rate. The rate of return earned on investment can be expressed as the cost of capital plus a premium and this return is multiplied by the rate of investment to establish income. For computing the EV/EBITDA ratio, factors for tax rates, depreciation rates, working capital and deferred taxes should be added. To demonstrate the process the P/E ratio is discussed first and the EV/EBITDA is subsequently addressed.

In modeling the P/E ratio, the drivers include:

- The growth rate in earnings in the short-run and the long-run
- The rate of return earned above the cost of capital in the short-run and the long-run
- The rate of inflation in the short-run and the long-run
- The real rate of interest
- The risk premium above the nominal interest rate in the short-run and the long-run

When deriving inputs for these value drivers, considerations include:

- The real growth rate in the long-run should not be more than the expected nominal growth rate in the economy (which some would say is the same as the real rate of interest)
- The spread between the return on equity and the cost of capital in the long-run should decline with increased competition and other factors; but it should not be zero. Without earning a return above the cost of capital, the company has no reason to be in business.
- Despite elegant theory on estimating the risk premium, a simple idea should drive the risk premium; when growth rates are lower and returns are lower, the risk is also lower. With stable growth and low earnings, the risk premium should not be much higher than credit spreads.

When applying the dividend discount model or the P/E equation, it is simple to assume that cash flow accruing to shareholders as dividends grow at different rates at different points in time. This idea of applying two or more growth rates in a DCF analysis is not new. The multi-stage approach can be applied to project a long-term rate of return relative to the cost of capital on a sustainable basis. To apply a multi-stage model, a long-term growth rate, a short-term growth rate and a transition period are assumed. The growth rate in the transition period is between the short-term growth rate and the long-term growth rate is given by the formulas below. After the formula is applied, the short-term growth rate gradually converges to the long-term growth.

In the case of using the CAGR to derive the transition factors, the number of years should be the number of transition years plus one. To see why this is so, consider a simple example where the transition period is one year. If the transition factor were computed without adding one, it would simply be the ratio of the long term amount to the short term amount. If this ratio were multiplied by the short-term amount, the transition year would contain the long term amount and not something in-between as illustrated below.

Transition Factor = Long-term/Short-term

Transition year amount = Short-term x Transition Factor = Short-term x Long-term/Short-term

On the other hand, when the transition factor is computed through adding a year to the transition period, then the transition factor correctly measures the CAGR because the transition adds another year to the process. Using the example with a short-term value of 10 and a long-term value of 15 along with a one year transition period, the transition factor would be:

Transition Factor = (Long-term/Short-term) ^(1/(years+1) = (15/10)^(1/2) = 1.225

Applying this transition factor yields the following series of numbers:

10   10 x 1.225=12.25   15

Growth Rate from Short-term to Long-term = (Long-term/Short-term) ^ (1/Transition Period+1) – 1

To see why you must add one to the transition period in the above formula, consider a case with a single transition period, or when the short-term period occurs in one year followed by one period of transition period and then the long-term period. In this case the transition period is defined as 1.0.

Transition Factor = (Long-term/Short-term) ^ (1/Transition Period+1)

and,

Transition Growth_t = Transition Growth_{t-1} x Transition Factor

Once computation of the transition factor is understood, the second idea is to create TRUE/FALSE switches for the different time periods – the short term period, the long-term period and the transition period. Switches for the short term period (period <= short-term) and the long term period (period > short-term + transition) are fairly straightforward to model as described in more detail in Chapter 2. The switch for the transition period could be a little more complex using the AND function. However one can make the process easier by noting the when the transition period exists, then both the short-term switch and the long-term switch are false. In every other period either the short-term switch or the long-term switch is true. This means that a simple test that the short-term switch equals the long-term switch can be applied to derive the transition factor switch (i.e. when both are false.)

Transition Factor Switch: Long-term Switch = Short-term Switch

Given values for the switches, the transition factors and the long and short-term parameters, the annual values can be computed using the following formula if the growth rate method for transition is used:

Annual Value = Short-term Value x Short-term Switch +
Long-term Value x Long-term Switch +
Last Year Value x Transition Factor x Transition Switch

Since this is a relatively long formula, you can use the ALT and ENTER function to put the various elements on different lines as shown above. The transition factors can be applied to:

- The short-term, transition and long-term inflation rate
- The short-term, transition and long-term real interest rate
- The short-term, transition and long-term risk premium
- The short-term, transition and long-term ROE versus Cost of Equity Spread
- The short-term, transition and long-term real growth rate in earnings

The final point to derive a model that explains the factors underlying the P/E ratio is to compute the implied dividend payout ratio that is consistent with the expected earnings growth. The dividend payout ratio can be computed as 1-g/ROE as explained above. Application of the interpolation techniques along with TRUE/FALSE switches to establish different variables is illustrated in the figure below:

Transition Factors											
Inflation Transition	**1.20**										
Risk Premium Transition	**0.87**										
Return above COC Transition	**0.83**										
Growth Rate Transition	**0.92**										
Model											
Period	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Short-term Switch		**TRUE**	**TRUE**	**TRUE**	**TRUE**	**TRUE**	**FALSE**	**FALSE**	**FALSE**	**FALSE**	**FALSE**
Long-term Switch		**FALSE**	**FALSE**	**FALSE**	**FALSE**	**FALSE**	**FALSE**	**FALSE**	**FALSE**	**FALSE**	**TRUE**
Transition Switch		**FALSE**	**FALSE**	**FALSE**	**FALSE**	**FALSE**	**TRUE**	**TRUE**	**TRUE**	**TRUE**	**FALSE**
Inflation Rate		**1.00%**	**1.00%**	**1.00%**	**1.00%**	**1.00%**	**1.20%**	**1.44%**	**1.73%**	**2.08%**	**2.50%**
Real Interest Rate		**1.50%**	**1.50%**	**1.50%**	**1.50%**	**1.50%**	**1.50%**	**1.50%**	**1.50%**	**1.50%**	**1.50%**
Nominal Interest Rate		**2.50%**	**2.50%**	**2.50%**	**2.50%**	**2.50%**	**2.70%**	**2.94%**	**3.23%**	**3.58%**	**4.00%**
Risk Premium		**4.00%**	**4.00%**	**4.00%**	**4.00%**	**4.00%**	**3.48%**	**3.03%**	**2.64%**	**2.30%**	**2.00%**
Total Cost of Capital		**6.50%**	**6.50%**	**6.50%**	**6.50%**	**6.50%**	**6.18%**	**5.97%**	**5.87%**	**5.88%**	**6.00%**
Risk Premium		**5.00%**	**5.00%**	**5.00%**	**5.00%**	**5.00%**	**4.16%**	**3.47%**	**2.89%**	**2.40%**	**2.00%**

After the implied dividend payout ratio is computed using the formula 1-g/ROE, the net income per share, the book value per share and the dividends per share can be calculated through creating a table of the progression in the balance of equity invested capital. The closing balance of the equity investment is the opening balance plus the income less the dividends. The income and dividends are computed from the following formulas:

- Net Income = Return on Equity (short-term, long-term or transition) x Opening Balance
- Dividends = Dividend Payout (short-term, long-term or transition) x Net Income
- Value of Equity = PV(at cost of cost of equity, of dividends)
- P/E Ratio = First Year Net Income/Value of Equity

The process of computing the P/E ratio using a build-up of the equity balance is illustrated below.

	P/E Ratio	19.90
Summary	Equity Value	25.37
	Market to Book	1.69

Growth Rates													
Real Growth Rate		3.00%	3.00%	3.00%	3.00%	3.00%	2.77%	2.55%	2.35%	2.17%	2.00%	2.00%	2.00%
Nominal Growth Rate		4.03%	4.03%	4.03%	4.03%	4.03%	4.00%	4.03%	4.13%	4.30%	4.55%	4.55%	4.55%
Dividend Payout Ratio		52.59%	52.59%	52.59%	52.59%	52.59%	49.51%	46.19%	42.63%	38.88%	35.00%	35.00%	35.00%
Book Value per Share Balance													
Opeining Book Value Per Share		15.00	15.60	16.23	16.89	17.57	18.28	19.01	19.77	20.59	21.47	22.45	23.47
Add: Earnings		1.28	1.33	1.38	1.44	1.49	1.45	1.42	1.42	1.45	1.50	1.57	1.64
Less: Dividends		0.67	0.70	0.73	0.75	0.79	0.72	0.66	0.61	0.56	0.53	0.55	0.58
Closing Balance	15.0	15.60	16.23	16.89	17.57	18.28	19.01	19.77	20.59	21.47	22.45	23.47	24.54
Valuation													
Discount Rate		6.50%	6.50%	6.50%	6.50%	6.50%	6.18%	5.97%	5.87%	5.88%	6.00%	6.00%	6.00%
PV Factor		0.94	0.88	0.83	0.78	0.73	0.70	0.67	0.63	0.60	0.56	0.53	0.50
Equity Value per Share	25.37												
P/E Ratio	19.90												

Using the EV/EBITDA Ratio in Terminal Value Calculations

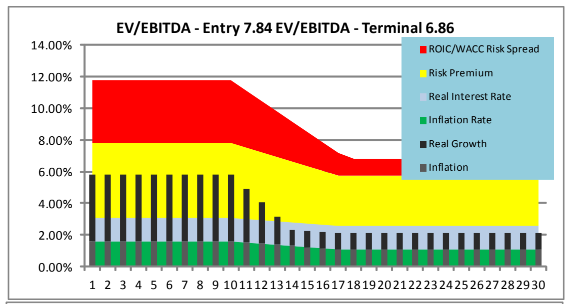
An alternative way to compute terminal value using in the DCF model is to apply a multiple to the terminal EBITDA rather than to use the standard terminal growth rate formula known as Gordon's model. This involves multiplying the EBITDA in the terminal year by an EV/EBITDA multiple. If this approach is applied, the terminal value is no longer directly sensitive to the discount rate and the growth rate as illustrated by the simplicity of the formula for terminal value:

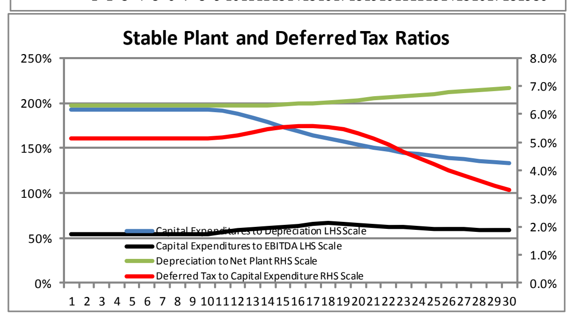
$$\text{Terminal Value}_t = \text{EV/EBITDA} \times \text{EBITDA}_t$$

The problem with applying the EV/EBITDA ratio in terminal value calculations is determining what multiple to use. If multiples are used from comparison of other companies – i.e. from relative valuation – a legitimate criticism is that multiples in the current market place are driven by short-term factors while the long-run multiples appropriate for use in DCF analysis are a function of stable growth rates and stable returns. Instead, the EV/EBITDA ratio can account for the return on investment, the cost of capital and the growth rate. This can be accomplished through simulating the theoretical multiple from assumptions with respect to cost of capital, rate of return, capital expenditure to depreciation relationships and the tax rate. The table and the chart below contain inputs used to compute the EV/EBITDA on an independent basis. The title of the chart shows that the terminal EV/EBITDA is 6.86 while the entry EV/EBITDA is 7.84. Different assumptions with respect to growth rates, returns, risks and transition factors yield different assumptions with respect to the terminal EV/EBITDA. Further, the assumptions for the risk premium – the most difficult assumption -- can be adjusted to make the current EV/EBITDA consistent with the ratio observed in the market.

Book Life (Asset Replacement)	25	
Tax Life	10	
Tax Rate	23.00%	
Working Capital to Plant	11.00%	
Transition Years - Growth	3	
Transition Years - ROIC	7	
Transition Years - WACC	6	
Short Term Period	10	

	Current	Long-term	
Inflation Rate	1.60%	1.10%	LT=ST
Real Interest Rate	1.50%	1.50%	
Risk Premium	4.70%	3.14%	LT=ST
Total WACC	7.97%	5.84%	
ROIC/WACC Spread	3.94%	1.06%	LT=ST
Total ROIC	12.22%	6.96%	
Real Growth Rate	4.20%	1.00%	LT=ST
Inflation Rate	1.60%	1.10%	
Nominal Growth Rate	5.87%	2.11%	
Stable Cap Exp/Depreciation	193.11%	160.22%	
Depreciation/Net Plant	6.30%	6.43%	
Capital Expenditures/EBITDA	54.72%	66.64%	
Deferred Taxes/Capital Expenditure	5.12%	5.54%	





Using the simulated EV/EBITDA ratio, sensitivity analysis with respect to different terminal periods, different fade periods and different assumptions with respect to the ability of a company to realize profits in excess of their cost of capital over long periods can be performed.

In developing the implied EV/EBITDA ratio rather than the implied P/E ratio from value drivers and transition periods, the process involves a table of net invested capital rather than the equity balance. As described in the previous part of the chapter, the invested capital can be evaluated using either a table of assets or a table of financing obligations including equity and net debt – recall that one can begin with common equity and net to compute invested capital or one can begin with net plant, goodwill, net working capital and deferred taxes. The process of computing the EV/EBITDA ratio works best using the asset side of the balance sheet rather than financing obligations. Net assets grow by capital expenditures and net working capital changes and the net assets are reduced by deferred tax changes. To demonstrate the mechanics of the process, the discussion begins by assuming no working capital or deferred taxes and that the factors listed below are known (derivation of each of the factors from the book life, the growth rate in assets, the tax life and the income tax rate):

- Return on Invested Capital
- Weighted Average Cost of Capital
- Capital Expenditures to Depreciation

- Depreciation Rate on Net Plant

The first step of the analysis is computing the investment balance as illustrated in the table below which is analogous to computing the balance of the equity capital. In the case of net investment, the balance decreases from depreciation expense and it increases from capital expenditures. The depreciation expense can be calculated from the opening balance multiplied by the net depreciation rate. Once the depreciation is known, the capital expenditures can derived from multiplying the amount by the ratio of capital expenditures to depreciation.

Investment Balance										
Investment	Prior Balance	1,000.00	1,030.00	1,060.90	1,092.73	1,125.51	1,159.27	1,194.05	1,229.87	1,266.77
Less: Depreciation	Inv x Rate	60.00	61.80	63.65	65.56	67.53	69.56	71.64	73.79	76.01
Plus: Cap Exp	Dep x Cap Exp	90.00	92.70	95.48	98.35	101.30	104.33	107.46	110.69	114.01
Ending Investment	Sum	1,030.00	1,060.90	1,092.73	1,125.51	1,159.27	1,194.05	1,229.87	1,266.77	1,304.77
ROIC	Input	12.0%	12.0%	12.0%	12.0%	12.0%	12.0%	12.0%	12.0%	12.0%
NOPLAT = EBIT x (1-t)	ROIC x Beg Bal	120.00	123.60	127.31	131.13	135.06	139.11	143.29	147.58	152.01
EBIT	NOPLAT/(1-t)	171.43	176.57	181.87	187.32	192.94	198.73	204.69	210.84	217.16
Taxes	EBIT x Taxes	51.43	52.97	54.56	56.20	57.88	59.62	61.41	63.25	65.15
Depreciation	Above	60.00	61.80	63.65	65.56	67.53	69.56	71.64	73.79	76.01
EBITDA	EBIT + Depreciation	231.43	238.37	245.52	252.89	260.47	268.29	276.34	284.63	293.17

Once the net investment balance is computed, the NOPLAT or net operating profit less adjusted taxes can be computed as the rate of return on invested capital multiplied by the opening balance of the investment. Because NOPLAT is equal to EBIT x (1-tax rate), the EBIT can be derived as the NOPLAT/(1-tax rate). After computing EBIT, the depreciation expense that has already been computed can be added to derive the EBITDA. With EBITDA, EBIT, the tax rate and capital expenditures, items for cash flow are available. The present value of cash flow is the enterprise value and the EV/EBITDA can be computed as illustrated below.

Computation of EBIT and EBITDA												
NOPLAT = (EBIT * (1-t)) = ROIC x Investment		115.0	114.3	113.9	113.8	114.1	103.1	94.7	88.7	85.0	83.8	86.0
EBIT = NOPLAT/(1-t)		176.9	175.8	175.2	175.1	175.5	158.6	145.7	136.5	130.8	128.9	132.3
EBITDA = EBIT + Depreciation		243.6	242.1	241.3	241.1	241.6	225.1	212.6	204.0	199.3	198.7	204.0
Ratio of Cap Exp to EBITDA		25%	26%	27%	28%	29%	32%	36%	40%	44%	49%	50%
Free Cash Flow												
EBITDA		243.6	242.1	241.3	241.1	241.6	225.1	212.6	204.0	199.3	198.7	204.0
Less: Taxes on EBIT		61.92	61.54	61.33	61.29	61.42	55.52	50.99	47.77	45.79	45.10	46.30
Less: Capital Expenditures		60.45	62.89	65.42	68.06	70.80	73.12	76.61	81.58	88.44	97.70	102.15
Free Cash Flow		121.22	117.65	114.51	111.76	109.39	96.42	84.98	74.66	65.07	55.85	55.50
Valuation												
Discount Rate		6.50%	6.50%	6.50%	6.50%	6.50%	6.18%	5.97%	5.87%	5.88%	6.00%	6.00%
PV Factor		0.94	0.88	0.83	0.78	0.73	0.70	0.67	0.63	0.60	0.56	0.53
Enterprise Value	1,992.68											
EV/EBITDA	8.18											

Subsequent discussion delves into the issues involving how to derive the depreciation rate as a percent of net plant and how to compute the ratio of capital expenditures to depreciation. In addition, the next section addresses how to add deferred taxes and working capital to the analysis. After including these items, the EV/EBITDA ratio can be derived from the lifetime of plant, the tax depreciation life and method, the growth rate in assets, and the ratio of working capital to net plant as illustrated on the above figure that shows the implied EV/EBITDA ratio assuming different drivers.

Stable Ratio of Capital Expenditures to Depreciation and Depreciation to Net Plant

The general question of how to compute capital expenditures in the DCF model is important in valuation, particularly in the terminal period. Some level of capital expenditures is needed to generate the EBITDA

consistent with the growth rate. The amount of capital expenditures required to generate future growth depends on the assumed return on investment as well as the lifetime of capital expenditures and the growth rate. If capital expenditures in the past few years have been relatively low and one assumes that the future capital expenditures will be consistent with the past, then the EBITDA may not be able to grow. On the other hand, if the capital expenditures have been high and they are assumed to continue, the EBITDA growth should be much higher. To understand the relationship between capital expenditures and depreciation, consider a case where a company does not grow in nominal terms. Here, the depreciation expense covers the amount of money that is required to replace assets and the capital expenditures equal the depreciation. If a company is growing, then the capital expenditures must be greater than the depreciation expense to cover both the replacement of assets and increases due to growth. This occurs even if the growth is only nominal growth resulting from inflation as the depreciation expense measures past expenditures and the capital expenditures are expressed in current currency.

To rectify the problem of assuming capital expenditures that are far too low or far too high in relation to EBITDA, analysts sometimes simply assume that a level of capital expenditures equal to depreciation will sustain the EBITDA growth. This means that in the terminal year, one can compute the capital expenditures directly from the depreciation expense. While this may seem to be too simple of a rule of thumb, it does have some logic. There are however a few problems with this method. First, if inflation and growth has occurred in the past, the depreciation expense can dramatically understate the required replacement capital expenditures because depreciation by definition lags the capital expenditure. Second, the timing of retirements of existing capital can affect the future requirements.

Using the notion that the growth requires a ratio of capital expenditures that is above one, the formula for prospective growth in assets can be expressed as follows:

Growth = Depreciation Rate on Net Plant x (Capital Expenditures/Depreciation – 1)

This formula is analogous to the growth rate formula for earnings that is driven by the retention rate. As with the retention rate formula, it is useful to re-arrange the formula and compute the ratio of capital expenditures to depreciation as a function of the depreciation rate and the growth rate. (Note that the depreciation rate is 1/life of the assets which means that the capital expenditure to depreciation is a function of the growth rate and the life of the assets):

Capital Expenditures/Depreciation = Growth/Net Depreciation Rate + 1

This formula implies that if the growth rate is zero, then the ratio of capital expenditures to depreciation is equal to unity. The longer the life, the higher the required capital expenditures to support growth as depreciation is relatively low because it includes older assets. The problem with the above analysis is that the net depreciation rate depends on the growth rate. If the growth rate is very high, the depreciation on net plant approaches the depreciation rate on gross plant, but if the growth rate is low, the depreciation rate on net plant is higher than the depreciation rate on net plant.

To compute the ratio of depreciation to net plant and the stable ratio of capital expenditures to depreciation that is not 100% as sometimes assumed in models because when assets grow, a simple life cycle model can be developed. The simple model works through the balance of investment including additions driven by growth and retirements that are a function of the asset life. The model can be extended indefinitely, but the ratio will stabilize at the end of the investment life. The example below shows a case where the ratio of capital expenditures to depreciation stabilizes at 122% rather than 100%.

When computing the ratio of capital expenditures to depreciation, one must first develop an assumption with respect to the growth of invested capital in plant and equipment. The ratio of capital expenditure to depreciation in perpetuity can computed once the growth rate in capital expenditures is input and retirements are computed from the depreciation life. Unfortunately, the calculations require some kind of assumption with respect to retirements of existing assets. (A simple assumption from depreciation rates can be made if a priori information is known.)

To model the future retirements, the first thing to note is that the retirements do not start until after the life of the plant. The first part of the formula should therefore be an IF statement involving the whether the year is greater than the life of the plant – before this year there are no retirements related to the capital expenditures. When the year is greater than the lifetime, then the retirements should begin. The retirements should look backward from the current year by the length of the life of the plant. This can be accomplished using the following formula:

Prospective Retirements = IF(Year > Asset Life, OFFSET(current cell,-1,-Asset Life))

With the investment balance computed, the depreciation expense can be computed. As with calculation of the net income in the P/E calculation above, the opening balance should be the basis of computing the depreciation. Once the depreciation is computed, the accumulated depreciation can be tabulated given the beginning balance of the accumulated depreciation. Then, the net investment balance is the gross investment balance less the accumulated depreciation. As with the stable ratio of capital expenditures to depreciation, other stable ratios include the ratio of depreciation to net plant, the ratio of capital expenditures to net plant and the ratio of capital expenditures to EBITDA.

Model												
Period	0	1	2	3	4	5	6	7	8	9	10	11
Growth Rates												
Real Growth Rate		3.00%	3.00%	3.00%	3.00%	3.00%	2.77%	2.55%	2.35%	2.17%	2.00%	2.00%
Nominal Growth Rate		4.03%	4.03%	4.03%	4.03%	4.03%	4.00%	4.03%	4.13%	4.30%	4.55%	4.55%
Investment Balance												
Gross Investment from Growth Rates	1,500.0	1,560.5	1,623.3	1,688.8	1,756.8	1,827.6	1,900.7	1,977.3	2,058.9	2,147.4	2,245.1	2,347.2
Implied Capital Expenditures		60.5	62.9	65.4	68.1	70.8	73.1	76.6	81.6	88.4	97.7	102.2
Depreciation Rate		6.67%	6.67%	6.67%	6.67%	6.67%	6.67%	6.67%	6.67%	6.67%	6.67%	6.67%
Depreciation Expense		66.7	66.3	66.0	66.0	66.1	66.4	66.9	67.5	68.5	69.8	71.7
Accumulated Depreciation	500.0	566.7	632.9	698.9	764.9	831.1	897.5	964.4	1,031.9	1,100.4	1,170.2	1,241.8
Net Investment	1,000.0	993.8	990.4	989.8	991.9	996.6	1,003.2	1,013.0	1,027.0	1,047.0	1,074.9	1,105.4

The stable ratio of capital expenditures to depreciation depends on the nominal growth rate in capital expenditures and the lifetime of the asset. With higher growth rates and the longer lived investments, the ratio of stable capital expenditures to depreciation is greater. The table below demonstrates how the ratio of capital expenditures to depreciation increases with both depreciation life and with the growth rate.

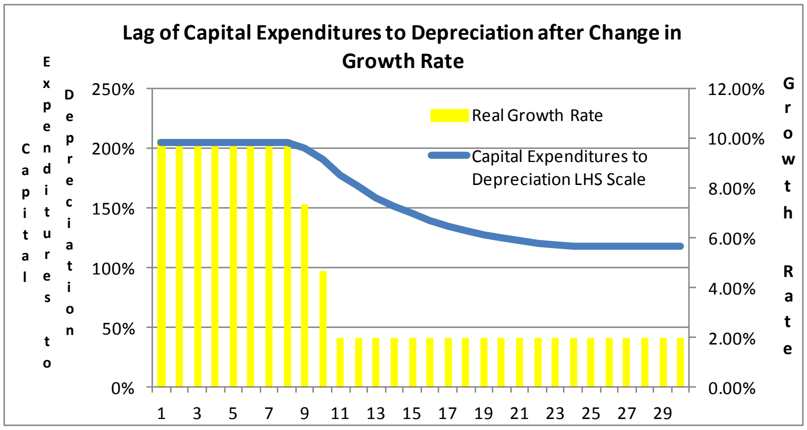
Ratio of Stable Capital Expenditures to Depreciation

		Growth Rate						
		0%	**1%**	**2%**	**3%**	**4%**	**5%**	**6%**
Investment Life	**3**	100%	102%	104%	106%	108%	110%	112%
	5	100%	103%	106%	109%	112%	115%	119%
	10	100%	106%	111%	117%	123%	130%	136%
	15	100%	108%	117%	126%	135%	145%	154%
	20	100%	111%	122%	134%	147%	160%	174%
	30	100%	116%	134%	153%	173%	195%	218%
	50	100%	128%	159%	194%	233%	274%	317%

Changing Ratio of Depreciation to Capital Expenditures to Depreciation with Changes in Growth

The above discussion provides the ratio of capital expenditures to depreciation if everything is in equilibrium. However if growth rate changes, the depreciation rate from the growth rate during the first period continues to have an effect on the ratio of capital expenditures to depreciation until all of the assets that were put in service are retired. For example, if the assets previously grew at a rate of 30%

implying a high ratio of capital expenditures to depreciation and then stop growing, the change in depreciation will lag the capital expenditure change until a full life cycle is worked through. The lag in capital expenditures to depreciation after growth rate changes is illustrated below where the ratio continues to decline for many years after the assumed change in growth. This means that one cannot simply insert the capital expenditures to depreciation into a DCF model as there will be a long-term transition period. The graph demonstrates that in the final year of growth in year eleven, the ratio of capital expenditures to depreciation has reached less than half of its progression moving from about 200% to about 100%. Similar phenomena occur for the ratio of depreciation to net plant.



In developing a comprehensive model of the EV/EBITDA the time lags until new stable ratios of capital expenditures to depreciation and stable ratios of depreciation to net plant. Method of working through issue one by one and using a comprehensive model that computes the EV/EBITDA short-term, transition and long-term periods. Describe how it is necessary to work through a life cycle – draw a diagram.

Need to first work through the asset and deferred tax balances to get capital expenditure to depreciation and see how there is a lag after the growth rate changes. Also get depreciation rate on net plant, change in deferred tax and deferred tax balance. Put in a graph of cycles.

Stable Ratio of Deferred Taxes to Capital Expenditures

Stable ratios for computing deferred taxes can be evaluated in a similar manner as the rate of capital expenditures to depreciation. Analysts tend to be a little intimidated by deferred taxes which sometimes results in large mechanical errors. With a bit of patience one can work through the deferred taxes and develop reasonable estimates for valuation. From an accounting standpoint, deferred tax represents a liability would be subtracted from enterprise value as the tax depreciation will be less than the book depreciation in the future. The accumulated deferred tax on the balance sheet contains the nominal value of the liability that would be repaid if there were no more capital expenditures or if the tax life equaled the book life. If the company keeps growing, the new capital expenditures will continue to generate new deferred taxes and a stable rate of new deferred taxes to can be computed in a similar manner as the ratio of capital expenditures to depreciation discussed above. As with book depreciation, an investment balance is set up for the deferred taxes instead of the book taxes. The difference between the tax deprecation and the book depreciation multiplied by the tax rate yields the change in deferred tax that should be added to free cash flow.

Ratio of Stable Deferred Tax to Capital Expenditures Assuming 40% Tax Rate and 20 Yr Life

	Growth Rate						
Tax Life	**0%**	**1%**	**2%**	**3%**	**4%**	**5%**	**6%**
1	0.00%	3.51%	6.51%	9.08%	11.28%	13.17%	14.80%
3	0.00%	3.12%	5.75%	7.96%	9.82%	11.39%	12.70%
5	0.00%	2.74%	5.00%	6.88%	8.43%	9.71%	10.76%
7	0.00%	2.36%	4.28%	5.85%	7.12%	8.14%	8.96%
10	0.00%	1.79%	3.23%	4.37%	5.26%	5.96%	6.50%
15	0.00%	0.88%	1.56%	2.08%	2.47%	2.75%	2.96%
20	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

The table implies that accumulated deferred taxes should be ignored in the valuation except to establish the base of future tax depreciation. The only case in which the accumulated deferred tax measures value is under the extreme case where there are no capital expenditures and where the weighted average cost of capital is zero. If the discount rate is positive, then the accumulated deferred tax underestimates the value of the future tax deductions. If the capital expenditures are positive and growing, then deferred taxes on the future capital expenditures offsets the liability from existing deferred taxes. For the analysis to be accurate, and estimate should be made of the exiting tax depreciation should be made. This can be established from the exiting balance of deferred tax and is illustrated in the case exercise.

The discussion above regarding the EV/EBITDA ratio includes an error from ignoring retirements of plant. The analysis assumes that capital expenditures are continually added to plant without ever being retired. This overstates depreciation expense (it does not overstate the net investment balance because retirements are removed from both the gross balance of investment and the accumulated depreciation.) To correct this problem one can include both retirements of existing plant and retirements of prospective plant that is added in the future. To model existing retirements, an existing retirement rate can be entered and formulas that test the remaining balance can be made. In modeling prospective retirements, the OFFSET command can be used so that different asset lives can be tested. The existing retirements could be the gross plant or the existing plant multiplied by the retirement rate. When modeling the existing retirements, the balance of existing plant should be tabulated (the existing plant less the accumulated retirements.) The retirements related to existing plant should stop when the existing net plant is negative and the final retirement should be the opening balance of the existing gross plant. To model the constraint that the accumulated existing retirements cannot exceed the existing plant, the MIN function can be used as follows:

Exiting Retirements = MIN(Existing Plant less Accumulated Retirements, Retirement Rate x Plant)

The retirement rate should be modeled by a balance of plant that increases or should be modeled in some other manner. If the plant has been increasing over time, the retirements should also increase over time. (One could establish a formula where a growth rate is input along with the last date of the retirements).

Stable Ratio of Working Capital to Net Plant and Other Items

When developing a discounted cash flow analysis, the terminal growth rate that is supposed to represent a stable period of equilibrium is generally lower than the growth rate during the explicit cash flow period (the explicit period and terminal period are defined above.) If revenues grow at a slower rate than the growth of working capital will also slow. However if one applies the standard perpetuity model using the formula:

Terminal Value = Final Year Cash Flow x (1+terminal growth)/(WACC – terminal growth),

a bias occurs. This bias arises because working capital changes in the final year cash flow are not consistent with working capital changes that would be present with a lower terminal growth rate. For example if the revenue growth is 50% in the final year cash flow and then falls to 3%, the terminal value will be understated. This occurs because the final year cash flow would subtract a large number for working capital changes, while the formula applied to compute value assumes a much lower revenue growth. To solve this problem one can create an additional stable period where the various different elements of cash flow are consistent with the stable growth rate assumption. This involves computing cash flows for a period t+1, rather than in the terminal period which may have higher growth rates.

A simple technique for taking care with respect to the terminal value is to add an additional period in the model after the explicit cash flow period – period t+1. Using this technique, assumptions with respect to the on-going capital expenditure, working capital, and deferred tax assumptions in perpetuity can be made that are all consistent with the long-term growth rates.

Error from Stable Working Capital Assumption

Working Capital as Percent of EBITDA	Explicit Growth Rate: 70%	50%	40%	30%	20%	10%	5%
50%	-24.49%	-20.41%	-18.50%	-16.88%	-15.92%	-16.54%	-18.19%
40%	-18.82%	-16.08%	-14.88%	-14.00%	-13.84%	-15.40%	-17.65%
30%	-13.70%	-12.10%	-11.51%	-11.29%	-11.86%	-14.30%	-17.13%
20%	-9.06%	-8.42%	-8.37%	-8.73%	-9.96%	-13.22%	-16.61%
10%	-4.84%	-5.02%	-5.44%	-6.32%	-8.14%	-12.17%	-16.10%
5%	-2.87%	-3.42%	-4.04%	-5.16%	-7.27%	-11.66%	-15.85%

Comparison of Implied Multiples from Alternative Methods

In setting up a model to derive the EV/EBITDA the first step after computing growth rates, returns and the cost of capital as described above is to recognize that in order to grow, the investment in plant must increase. For example, if a manufacturing firm is to increase sales by 5% and it is operating at full capacity, then the capacity must increase 5%. If capacity increases by 5% and the price of adding capacity remains constant, then the gross investment balance must also increase by 5%. To model investments, the beginning gross investment balance must be entered. If the model does not explicitly account for retirements, the construction expenditures are the difference between the beginning and ending balance of the gross investment.

The drivers of the EV/EBITDA ratio include growth rates, returns, risk premiums, the tax rate and the inflation rate. Instead of using return on equity and cost of equity as in the P/E discussion above, the EV/EBITDA is driven by the return on invested capital and the weighted average cost of capital. Computation of the EV/EBITDA could be made with and without an adjustment for retirements. Differences between the analysis of EV/EBITDA and the P/E ratio involve taxes, depreciation and net investment rather than equity. Once growth rates, transaction factors, returns and cost of capital are computed, differences involve computation of income taxes, depreciation, capital expenditures, EBITDA and free cash flow.

Book Life (Asset Replacement)	25
Tax Life	10
Tax Rate	23.00%
Working Capital to Plant	11.00%
Transition Years - Growth	11
Transition Years - ROIC	7
Transition Years - WACC	5
Short Term Period	7

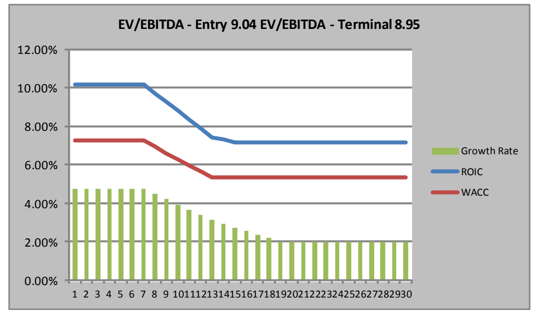
Sensitivity

LT=ST

Reset

	Current	Long-term	
ROIC	10.19%	7.18%	LT=ST
WACC	7.25%	5.33%	LT=ST
Growth	4.75%	2.01%	LT=ST

Year of Valuation	1	7
EV/EBITDA - Theoretical	**9.04**	**8.95**
EV/EBITDA - Value Driver	11.02	12.17
EV/EBITDA - Growth Rate	11.02	11.00



Regression Analysis of P/E, EV/EBITDA, Market to Book Ratio

To be completed.

Bridge between Enterprise Value and Equity Value and between Invested Capital and Equity Capital

This section discusses practical issues in measuring the difference between equity value and enterprise value discussed above when introducing the discounted cash flow (where the only difference was assumed to be net debt.) Liabilities or assets other than net debt create a difference between enterprise value (measured as the present value of free cash flow) and the equity value that actually accrues to shareholders. This difference between equity and enterprise value defined below as the bridge arises in situations other than net debt when computing value with the discounted cash flow method. One other instance is computing enterprise value from market capitalization for purposes of calculating the EV/EBITDA ratio used to establish value from comparable companies. Another case is computing the invested capital from the common equity investment on the balance sheet when computing the return on invested capital. A third case is computing the implied enterprise value in an acquisition given the consideration paid for the common shares of a target company. Items that should be included in the bridge vary depending on objectives of the computation. In order to determine which items should be included in the bridge between enterprise value and equity value or between invested capital and equity capital, it is important to understand what is included in free cash flow and how the various assets and liabilities affect cash flow to equity holders. While it may seem mundane and relatively simple to establish asset and liability items that should be included in the bridge, the process is in fact tricky and depends on the problem being addressed. Because of this, separate sections address the bridge between equity value and asset value below.

Equity Value from Enterprise Value when Using the Discounted Cash Flow Model

When computing equity value and enterprise value in the discounted cash flow model, a number of adjustments may be required for various assets and liabilities that are not directly or indirectly included in free cash flow and the enterprise value. The central issue in assessing whether an adjustment should be made in various instances is to work through each item on the balance and determine the following:

1. Does the asset produce cash flow to equity owners that is not directly or indirectly already measured in the free cash flow that is used to measure enterprise value;

2. Does the liability result in cash outflow made by equity owners that is not directly or indirectly measured in free cash flow and enterprise value;

If the above conditions are met, the item should be included in the bridge between computed enterprise value and equity value. In valuation analysis, the amount of the asset or liability adjustment should be the present value of the cash flow not included in the enterprise value that could theoretically be realized by shareholders. This can often be established in theory by pretending that the asset or the liability could be sold at the date of the asset valuation.

To demonstrate how various values are determined, a very simple hypothetical company is used to guide some of the discussion. This company simply purchases risk free treasury bonds and then distributes proceeds from the bond redemption and from the interest on treasury bonds to its common shareholders as dividends. As with many companies, the very simple business became messy because of historic acquisitions and questionable decisions made by its management. Because of attempting to save costs through buying an old building that has caused local protesters to mobilize, the company has been embroiled in various different legal battles. Further in borrowing money to buy treasury bonds, the company has not matched the repayment timing of the borrowing with the duration of cash flows that will be realized from the cash proceeds. Other than the basic business, the company has the following:

- Some extra cash on the balance sheet that produces an little income that is not included in free cash flow
- A loan (recorded as a note receivable) to a sister company that is due in three years
- A long-term account receivable that will finally be received as cash by the company in two years which was accompanied by higher revenues for not receiving cash on a timely basis
- An office building that is an eye sore and will have to be dismantled in five years the cost of the dismantling is included as a liability on the balance sheet.
- A potential liability related to lawsuits on the office building that is not recorded on the balance sheet
- Counter lawsuits against the city for condemning the building that could generate cash flow
- A few miscellaneous investments, the income of which is not included in free cash flow
- A pre-paid expense to a lawyer related to the lawsuits
- A parcel of land that is not currently producing income which will replace the current building
- Another parcel of land that has been purchased as plan B, in case the preferred building cannot be constructed
- Accounts receivable related to interest income that has been recorded in-between the semi-annual dates at which payments are received.
- Accounts payable to employees reflecting the fact that the employees are paid only once every three months.
- A long-term accounts payable to one of the legal firms as work has been done, but payments will not be made until the court decision is made.
- Because taxes are recorded on the books when interest is accrued, but taxes are paid later on when payments are received and quarterly tax returns are prepared, a deferred tax liability is recorded on the books.
- The company has been in existence for many years and it has a defined benefit plan which means that former retired employees will receive fixed pre-determined amounts in the future. The company thought it had enough cash in a trust fund to meet the present value of future obligations, but the trust fund assets were invested in stocks that took a big tumble during the financial crisis and it is now not big enough to meet the future obligations.
- The company generally invests in long-term securities that have maturities ranging from two to five years. However it borrows using short-term loans. Because of exposure to increasing interest rates, the company uses swaps to fix some of its interest rate exposure. The company entered into interest rate swaps when the fixed swap rate was high. If the company were to try and sell the interest rate swap, it would realize a loss because other companies can enter into a swap contract at a lower rate. The amount of this loss is recorded as a fair value of derivative liability on the balance sheet.

- The company is separated into ten different subsidiary companies. One of these companies is owned in part by a joint venture partner. This company has performed worse than any of the other subsidiary companies and the investment of the joint venture partner is included as minority interest on the balance sheet.
- The company was purchased from another company three years ago. Due to purchase accounting, the company recorded goodwill and increase in plant value and intangible assets related to customer lists on its books. In addition, a large deferred tax liability was recorded to reflect the fact that the intangible assets and the increase in plant value that were booked during the acquisition.
- The company pays many of its employees through stock-based compensation rather than through cash.

In computing the enterprise value, it is assumed that the company carefully computes the opportunity cost for investing in treasury securities that have different tenors (the company uses interest income in EBITDA which would not be consistent with the rule written in many textbooks, but this is the fundamental business of the company). As the cost of capital is the return that could be realized on investments with similar risk, the company computes the rate of return that could be earned on each tranche of the treasury securities and then it weights the average. For example, if the company only owned two treasury bonds, and one of the bonds had a long-term maturity of thirty years and the other bond had a maturity of two years, then the weighted average cost of capital should be the current return that could be earned by an investor in the thirty year bond and the two year bond. The weighting would in turn depend on the market value of the two bonds. This is the definition of cost of capital and for once it is not ambiguous.

The example is intended to illustrate many of the balance sheet items and contingent assets and liabilities that may or may not be included in the bridge between equity value and enterprise value. Through assuming the company invests (probably on an inefficient basis) in treasury bonds, the weighted cost of capital should be less ambiguous than for a company investing in other types of real assets.

Case 1: Bridge between Enterprise Value and Equity Value when Applying DCF Calculation

The first situation discussed is what items to include in the bridge between enterprise value and equity value when computing value using the discounted cash flow model. A number of different items are discussed below that affect both the bridge between equity value and enterprise value and the weighted average cost of capital. In considering each item, one should first evaluate whether the value of the item is already incorporated in enterprise value and in calculation of free cash flow. The manner in which each of the items should or should not be included in the bridge between enterprise value and equity value is addressed below through determining how the item is captured in the future cash inflow or outflow of the company. If the item is not included in cash flow and enterprise value and it affects the value of the company to equity investors, it should be included in the bridge between equity value and enterprise value. Investments in plant, goodwill and intangible assets do not generally produce income that is not already incorporated in free cash flow. As such the value of these investments should not be included as a separate item in the bridge between enterprise value and equity value. Other items on the balance sheet described below may or may not affect both the bridge between equity value and enterprise value and the WACC.

Surplus Cash

The first item addressed is the surplus cash. Surplus cash is defined as the extra cash that is not required to be held in the company to run day to day operations. Surplus cash is supposed to be amounts held as cash and short-term investments that are above the cash that is necessary for providing liquidity to meet day to day operations. Surplus cash is often computed on an approximate basis by assuming that operating cash on hand is 2% of total revenues. It is common to subtract surplus cash from debt and label the result as net debt because surplus cash could theoretically be used to repay debt.

As explained below, the distinguishing between operating cash and surplus cash is ambiguous and one can argue that operating cash should be treated in exactly the same manner.

Surplus cash earns interest and it eventually generates future cash flow from redeeming the cash (even if the interest rate is minimal). The cash and interest can eventually be distributed to equity holders as cash flow. Surplus cash clearly meets the criteria which qualifies it as part of the bridge between enterprise value and equity value. First, surplus cash can be redeemed to equity investors and, second this cash is not included in the computation of free cash flow used to establish enterprise value. However, even this seemingly simple item raises questions. Many investment banks include net debt rather than gross debt in the capital structure when computing the cost of capital as illustrated in the excerpt below. However in our example, the weighted average cost of capital has been carefully constructed to reflect the risks of assets that are represented in free cash flow and enterprise value (i.e. the alternative rate of return that could be earned on identical treasury bonds). In this example, including net debt in the cost of capital would distort the cost of capital because the cost of capital already measures alternative returns that can be earned for similar assets. This begs the question of whether surplus cash should be included in the weighted average cost of capital in other instances. Consider for instance the case of Microsoft Corporation that had US$ 44 billion of cash on its balance sheet at the end of 2010. If the cost of capital for Microsoft is computed through making a comparison with other companies such as Apple which also has a lot of cash and if the beta is de-levered and then re-levered with the surplus cash (implying an equity to capital ratio of more than one), then the surplus cash should be in the weighted average cost of capital calculation because it is incorporated in the beta.

If the beta and the cost of capital are somehow measured without re-leveraging for the net debt and the asset beta is used in the cost of capital, then the net debt should not be part of the weighted average cost of capital. Stated differently in a very extreme example, pretend that a company with a very risky activity also had massive amounts of cash on its balance sheet that dwarf the risky activity. In traded markets, the company appears to have a low risk. If this low risk is used in measuring enterprise value, then the WACC calculation should also be adjusted to correct the distortion created by the surplus cash. Here, through subtracting the surplus cash from the other capital supporting the assets, the remaining overall WACC is increased. The key in both putting something in the weighted cost of capital calculation and the bridge is consistency.

WACC Analysis

Company	Levered [1] Beta B_L	Net Debt	D/E	% Debt	Effective Tax Rate	Unlevered Beta B_U
ITC Holdings Corp.	0.92	$2,264	0.97	49.2%	38.1%	0.58
Cleco Corporation	0.75	1,161	0.81	44.8%	15.3%	0.44
UniSource Energy Corporation	0.77	1,838	1.87	65.1%	55.0%	0.42
ALLETE, Inc.	0.76	559	0.56	36.0%	34.3%	0.55
NorthWestern Corporation	0.82	855	0.98	49.4%	37.3%	0.51
MGE Energy, Inc.	0.71	365	0.43	30.3%	35.5%	0.55
CH Energy Group, Inc.	0.79	453	0.58	36.5%	37.6%	0.58
UIL Holdings Corporation	0.88	764	1.05	51.1%	41.8%	0.55
El Paso Electric Company	0.79	740	1.09	52.1%	32.8%	0.46
Empire District Electric Company	0.77	746	1.17	54.0%	32.5%	0.43
Unitil Corporation	0.54	325	1.45	59.1%	32.0%	0.27
Central Vermont Public Service Corporation	0.74	176	0.80	44.4%	39.6%	0.50
Florida Public Utilities Company	0.73	49	0.56	36.0%	35.0%	0.53
Average	**0.77**		**0.95**	**46.8%**	**35.9%**	**0.49**

Cost of Equity Build Up	
Unlevered Beta	0.49
Levered Beta	0.65
Risk Free Rate (Rf) [2]	3.6%
Market Risk Premium	5.0%
Cost of Equity	**6.9%**

Assumed Target Capital Structure % [4]		Pre-Tax Cost	After-Tax Cost	WACC Build-Up
Net Debt	32.4%	5.6%	3.9%	1.3%
Equity at Market Value	67.6%	6.9%	6.9%	4.6%
Total	100.0%			
Weighted Average Cost of Capital (WACC)				**5.9%**

Inputs	
Date	07/30/09
Beta Update	07/31/09
Debt Rates	07/30/09
t = Tax Rate	30.9%
MRP = Market Risk Premium	5.0%
Risk Free Rate	3.6%
Assumed Credit Spread	2.0%

WACC Sensitivity

Beta	% Debt 20.0%	27.4%	32.4%	37.4%	40.0%
0.39	5.5%	5.5%	5.5%	5.4%	5.4%
0.44	5.7%	5.7%	5.7%	5.7%	5.6%
0.49	6.0%	5.9%	5.9%	5.9%	5.9%
0.54	6.2%	6.2%	6.1%	6.1%	6.1%
0.59	6.4%	6.4%	6.4%	6.3%	6.3%

Formulas

Levered Beta (B_L)= Unlevered Beta (B_U) * (1+ D / E) * (1 - t)
Unlevered Beta (B_U)= Levered Beta (B_L) / (1+ D / E) * (1 - t)
Cost of Equity (Re) = Rf + Beta * MRP
WACC = ((D / (D + E) * Rd * (1 - t)) + (E / (D + E) * Re
Adjusted Beta = Unadjusted Beta * 0.67 + 1 * 0.33

While it is generally taken for granted that operating cash should be differentiated from surplus cash (at least in theory), this issue is not obvious. The idea behind excluding operating cash from the bridge is that some level of operating cash is necessary to run the business and that free cash flow could not be generated without the operating cash. However, an argument can be made that operating cash should also be included in the bridge between enterprise value and equity value. Assume there is a high interest income rate earned on cash held in a bank of 10%. The interest income would generally not be included in EBITDA, free cash flow or enterprise value. This interest income clearly has value to equity investors and that value would be nowhere in the DCF valuation if the operating cash is not included in bridge between enterprise value and equity value. The treatment of operating cash is one of the many things in finance that is taken for granted but is ambiguous. The argument that cash is necessary to generate the EBITDA is not much different than saying an overdraft short-term debt facility is necessary to run the business. It would be standard practice to deduct borrowings from the overdraft facility from the enterprise value.

Operating cash is a by-product of operations.

Discuss extreme example with very high cash balance.

Short-term Accounts Receivable and Inventory

One could argue that accounts receivable and inventory other related current assets are not very different than surplus cash. If the company would stop operating today, then these items would produce cash flow for shareholders just like the surplus cash. However, there are key differences between these current asset items and the surplus cash. First, the implicit finance cost of accounts receivable and inventory is incorporated in EBITDA and enterprise value. If a company demands faster payment to reduce accounts receivable balances, the reduction in these balances will probably reduce revenues. Alternatively, if a company is more generous with payment terms, revenues and EBITDA should increase implying that the financing effects of accounts receivable are already in cash flow. Similarly in the case of inventories, the company could reduce its inventory, but may have to increase the cost of goods sold implying that financing effects of the inventories are included in the EBITDA. Second, unlike surplus cash, the accounts receivable items are necessary in order to produce the free cash flow and are therefore incorporated in enterprise value. Third, as long as the firm is in existence and growing, the cash flow from accounts receivable and inventories will never be realized as cash flow to equity investors because previous accounts receivable are replaced with new accounts receivable and old inventories are replaced with new inventories. Finally, as the replacement of the accounts receivable and the replacement of inventories is directly included in the changes in working capital in the cash flow calculation it should not be part of the bridge between equity value and enterprise value. This contrast between surplus cash and accounts receivable, while relatively obvious provides a foundation for discussing other items.

Long-term and Short-term Notes Receivable and Long-term Accounts Receivable

A company may provide loans to other associated companies or to non-associated companies. These loans should produce some kind of interest income. If the loan interest is not included in the free cash flow that establishes enterprise value then the notes receivable, whether classified as long-term notes or short-term notes should be part of the bridge between enterprise value and equity value. When the notes are redeemed, cash flow will be realized by investors and it should not be part of free cash flow. If the notes receivable are made at below market rates of interest, then the value of the notes included in the bridge should be lower than the amount that is recorded on the balance sheet. In an extreme case, if the notes are very long-term and/or they are continually re-issued after coming due and they have no interest rate, the market value of the notes would be zero. As with other items, it is the market value of the notes rather than the book value that should be included in the bridge between enterprise value and equity value when applying the DCF model.

A long-term account receivable should in general in theory be similar to a notes receivable and the timing of when the receivable is not relevant in valuation. The treatment of a long-term receivable depends if it is a one-off item and it is not directly included in the free cash flow used to compute enterprise value. If the long-term accounts receivable is not expected to recur and if the redemption of the accounts receivable is not already included in free cash flow (as an increase from lower working capital), then it should be included in the bridge between equity value and enterprise value. Otherwise redemption of the long-term receivable is included in the free cash flow no adjustment is necessary.

The manner in which the notes receivable and other investments should be included in the WACC calculation depends on the way in which discount rates are adjusted for the notes receivable. For example, when betas de-leveraged are re-leveraged, if the notes receivable are included in the process of de-levering betas, then the notes payable should be included as a negative number in the WACC calculation. In this case, the cost of capital associated with the notes receivable should be the rate of return that could be realized from lending to the company in given its credit quality and the terms of the loan. The example below illustrates how a company with an asset beta of 1.30 implies that the measured equity beta is 1.06 because of the notes receivable. To make the adjustment to WACC, the notes receivable should be included as a negative item in the WACC in order to appropriately measure the cost of capital associated with free cash flow.

		Market Value	Beta	
Notes Receivable		200	-	
Other Assets Generating Free Cash Flow		900	1.30	
Total Assets		1100		
Equity		1100		
Cost of Capital for Assets and Equity			Equity	Assets
Beta			1.06	1.30
Risk Free Rate			4.00%	4.00%
Equity Market Risk Premium			5.00%	5.00%
Equity Cost of Capital			9.32%	10.50%
WACC	Amount	Percent	Cost	WACC
Notes Receivable	-200	-22%	4.00%	-0.89%
Equity	1100	122%	9.32%	11.39%
Total	900			10.50%

Long-term and Short-term Prepaid Expenses

In the example above, a pre-payment was assumed to be made to a lawyer for expenses that have not yet been incurred. If the expense – e.g. the trial – occurs within a year, then the pre-paid expense would be classified as current asset, while if the expense occurred after a year the pre-paid expense would be classified as a long-term asset. If the expense associated with the trial is included as a cash outflow in the EBITDA, free cash flow and enterprise value calculation (even though it will not really be paid), then the prepaid expense should be added to the bridge between equity capital and enterprise value. This is because the expense included in the free cash flow that makes the cash flow seem lower because of the cash flow to the lawyer that will not actually be paid. As the enterprise value is artificially lowered by the expense that is not actually a cash outflow, a correction should be made and the extra cash flow from prepayment should be somewhere in the valuation. The place to put this correction is in the bridge between equity value and enterprise value. (In theory the pre-paid expense should be valued as the present value of the expense.) On the other hand, if the legal for the trial is not separately included in the free cash flow because it has already been paid, then the pre-paid asset should not be included in the bridge. If the prepaid expense is in cash flow, the cash flow and the enterprise value correctly measures the enterprise value and no adjustment is necessary. The treatment of prepaid expenses demonstrates the importance of understanding what is implicitly or explicitly in the cash flow. If the pre-paid expense is related to taxes or other items that occur on an on-going basis, then the cash flows generally include the expense as an assumed cash outflow, but the pre-paid expense must be repaid as the previous expense is made. With a growing company this asset is never realized as cash by the equity investors even if the full expense is included in cash flow. Therefore, the prepaid expense should not be included in the bridge similar to the case with accounts receivable. In summary, current pre-paid expenses should not be included in the bridge and long-term one-off prepaid expenses should only be included at the present value of the expense if the expense is not included as an operating expense in the cash flow.

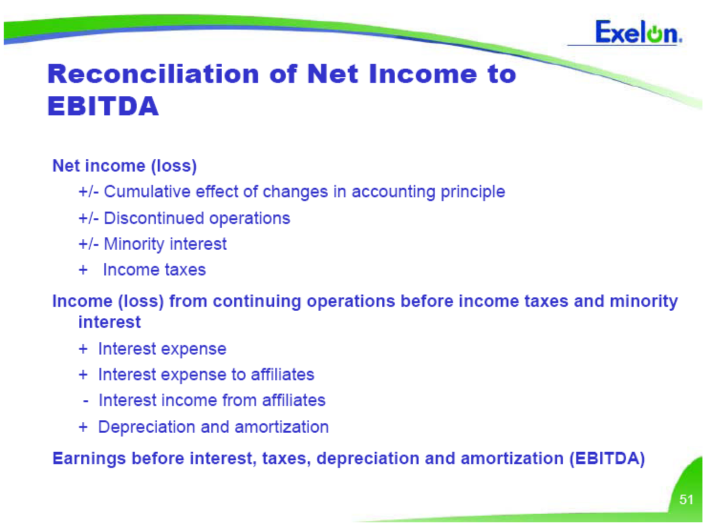
On the liability side of the balance sheet, the principle is similar for deferred expenses, except that payments have not yet been made and the accrued expense has occurred. For example, resource companies may have an account named accrued reclamation costs to measure the future expenditures that will be incurred for cleaning up a site after its productive life. If cash flow associated with this future liability is not included in free cash flow (for example as a future capital expenditure or operating expense) then the item does reduce value for equity investors. In this case, the present value of future expenditures should be included as an adjustment to enterprise value in computing the equity value. On the other hand, if the future expenditures are included as part of cash flow (probably many years into the future), the liability is already incorporated in free cash flow and enterprise value implying that no adjustment should be made.

Land and Other Assets not Currently Producing Cash Flow

Finance texts often describe the example of land that is held for building a future factory or a real estate development as something that should be included in the bridge between enterprise value and equity value. If the land is not currently earning income, but could be sold at a substantial sum, the additional amount should be included in the bridge between enterprise value and equity value. For example, suppose a parcel of land is currently earning nominal amounts for being used as farmland but that the land will be used in five years for development of a large commercial building. If the farm income is included in the free cash flow, then difference between the value that the land could be sold for and the present value of the farm land should be included in the bridge. If one takes this example to the extreme – say a hotel company buys land all over the world that may or may not be used in developing hotels, then the beta of the company should incorporate the risks associated with land speculation and the value of the land should be included as an element in the weighted average cost of capital calculation with a high beta. This adjustment to the WACC would probably never really be made, but it illustrates how to think through the problem. Because this example is used by textbooks it sometimes causes confusion in practical applications. For instance the value of land upon which a factory sits may could be argued to be an additional item that should be included in the bridge between equity value and enterprise value. Unless the factory is planned to be sold in a short period and the cash flow from the sale is not included in the flow, this adjustment is generally not appropriate.

Other Non-Associated Investments, Discontinued Operations and Long-term Investments

Long-term investments for which the income is not included in free cash flow should be included in the bridge between enterprise value and equity value as should generally be treated in the same way as the surplus cash. The fact that some investments are due in one year and others are expected to be held for longer periods makes no difference as to whether the investments should be included in the bridge. An exception to this is if the investments are held to pay a future liability such as the decommissioning of a nuclear plant. In this case either both the liability and the investment should be included or neither the liability nor the investment should be part of the bridge between enterprise value and equity value. Investments in non-associated companies and other investments should be treated in the same way as the long-term investments and the surplus cash as long as the income associated with these investments is not included in EBITDA and the free cash flow. On the other hand, if the income from other non-associated investments is included in the free cash flow and therefore included in the enterprise value, it would clearly be double counting to also include the investments as a bridge between enterprise value and equity value. The computation of EBITDA is not consistent among analysts, which means treatment in the bridge can be different. Sometimes calculation of EBITDA begin with revenues and cash expenses or begin with operating income and add back depreciation expense thereby excluding income from non-associated investments. This approach means that EBITDA, free cash flow and enterprise value concentrates on the business activities of the firm and excludes equity income from non-associated investments. Here, as income and cash flow from non-associated investments is not included in enterprise value, and it follows that the value should be included in somewhere else; namely in the bridge between equity value and enterprise value. On the other hand, some analysts compute EBITDA literally which means that one begins with earnings and then adds back depreciation, amortization, total net interest expense and taxes as shown below. In this case, income from non-associated investments is implicitly included in EBITDA, free cash flow and enterprise value. The treatment of non-associated investments in the bridge between enterprise value and equity value depends on the way the income is treated. If non-associated income is included in EBITDA, free cash flow and enterprise value, then In the example above, a pre-payment was assumed to be made to a lawyer for expenses that have not yet been incurred. If the expense – e.g. the trial – occurs within a year, then the pre-paid expense would be



The treatment of assets, income and losses that are classified as discontinued operations is generally analogous to income from non-associated investments where the non-associated investments are not included in EBITDA, free cash flow and enterprise value. The income or losses from discontinued operations should not be included in EBITDA and the value of the discontinued operations should be at market value. If the value of discontinued operations is included in the bridge between equity cash flow and enterprise value, then the cash flow forecasts should not include proceeds from the sale of the discontinued operations. Instead, the discontinued operations should simply be stuck on the balance sheet in the forecasts. As with surplus cash, if the cost of capital is computed through either directly or indirectly accounting for the risk of non-associated investments, long-term investments and discontinued operations should in theory be included as a negative item in the WACC. Assume that the discontinued operations were a very risky activity and the continuing operations have low risk. When using historic volatility to gauge the cost of capital and the risk, the free cash flow will have a low value if no adjustment is made in the WACC for the higher risk discontinued operations. The manner in which discontinued operations should theoretically be adjusted in the WACC is illustrated in the table below. In this case, the WACC estimated from historic analysis of the stock including discontinued operations is assumed to be 11.6%. Through somehow knowing the cost of capital of discontinued operations is 20%, one can derive the cost of capital of 6% that should be applied to free cash flow from continued operations. In theory similar adjustments should be made for long-term investments and for equity investments in non-associated companies.

WACC Adjustments for Discontinued Operations

Cost of Capital Assumptions				
Cost of Capital for Continuing Operations	6%			
Cost of Capital for Discontinued Operations	20%			
Value Assumptions				
Value of Discontinued Operations	200			
Total Equity Value	500			
Weighted Average Cost of Capital - Historic	Value	Percent	Cost	WACC
Discontinued Operations	200	40%	20%	8.00%
Continuing Operations	300	60%	6%	3.60%
Total	500			11.60%
Adjusted WACC for Continuing Operation	Value	Percent	Cost	WACC
Equity Value	500	167%	11.60%	19.33%
Less: Discontinued Operations	-200	-67%	20.0%	-13.33%
WACC for Contining Operations	300			6.00%

Debt, Debt Issuance Costs, Derivative Assets and Liabilities Associated with Hedging Interest Rates

Debt is generally the primary item in the bridge between enterprise value and equity value as the interest rate and repayment of debt is not included in free cash flow and enterprise value, but it must be incurred by equity investors. The primary question related to debt is how to make a valuation of debt for purposes of the bridge. Issues associated with valuing debt include (1) whether market value or book value should be used; (2) if debt issuance costs (an asset on the balance sheet) should be deducted from the debt balance; (3) how should derivative assets and liabilities associated with the valuation of interest rate swaps and exchange rate hedges be included in the bridge between equity value and enterprise value; 4) should adjustments be made for prospective changes in the capital structure; and, (5) how should the value of debt be adjusted if an acquisition changes the risk associated with the debt. The most precise way to account for debt would be to measure the future cash flow obligations that arise from the debt and value those obligations at the opportunity cost of debt. The opportunity cost of debt should in theory be measured using a discount rate that reflects the risk associated with probability of default and loss given default associated with the free cash flow. As the option value of lending increases with more debt in the capital structure, the option value of equity should decline and in theory the overall cost of capital should not change. If loans have been made at different fixed rates than the opportunity cost of debt capital (which includes debt fees paid to bankers and bond underwriters), then this difference in value should be included in the bridge between equity value and enterprise value. If debt is traded in the market, the value of the debt should reflect the opportunity costs of lending at current rates and it should be used in the bridge. However, the market value should be reduced to account for arrangement fees that must be paid by equity holders as new debt is issued to finance on-going capital expenditures of the company.

Through application of the consistent principle that cash flows not included in EBITDA, free cash flow and enterprise value, the market value of debt has similar characteristics to the other items above. The cost of capital associated with the market value of the debt should be measured at the date of the debt valuation and be measured in the WACC so that the cost of capital associated with the enterprise value is not distorted. This means that if one believes in computing asset betas and then re-levering the betas (which is demonstrated to be inaccurate below) that the re-levered beta must be consistent with the market value of the net debt at the valuation date. For example, it is inappropriate to use a target debt to capital ratio that is inconsistent with the net debt used in the equity to enterprise value bridge. If for example, the debt actual value of the debt results in a debt to capital ratio of 70%, but the target debt to capital ratio is 40%, the valuation would be distorted through use of the target capital structure in the WACC and the re-levered beta, but use of the actual market value of debt in the bridge. The problem of inconsistent valuation is illustrated in the example below.

Distortions from Target Capital Structure		
Assumptions		
Unlevered Beta	0.8	
Target Debt to Capital	40%	
Actual Market Value of Debt to Capital	70%	
Risk Free Rate	4.50%	
Equity Market Risk Premium	5.00%	
Computed Cost of Capital		
Cost of Equity Using Target Debt to Capital	11.17%	
Cost of Equity Using Measured Debt to Capital	17.83%	
Theoretical WACC using Asset Beta	8.50%	
Current Values		
Equity Value	2,000.00	30%
Market Value of Debt	4,666.67	70%
Total Market Value of Enterprise	6,666.67	
Contining Cash Flow		
Assumed Perpetual Cash Flow	566.67	
Perpetual Debt Service at Risk Free Rate	210.00	
Perpetual Equity Cash Flow	356.67	
Implied Valuations		
Value of Equity using Acutal Debt Ratio	2,000.00	
Value of Equity using Target Debt to Capital	3,194.03	
Enterprise Value	6,666.67	
Less: Market Value of Debt	4,666.67	
Value of Equity	2,000.00	

In practice, computing the market value of each separate debt issue where differences in credit spreads for debt issues with different durations; differences in the yield structure of different debt issues; reflection of interest rate swaps for each issue; and inclusion of debt fees in the cost of debt capital is not practical. Instead, if the interest rate on most debt issues is reasonably similar to the interest rate that would be incurred on new debt issues or if most debt issues have variable interest rates, then the market value of debt is assumed to be approximately the same as the value of debt on the balance sheet. If this assumption that the market value equals the book value is made, then it is appropriate to make other adjustments in the bridge between equity value and enterprise value. The first adjustment is that debt issuance costs should be included as a reduction in the value of debt which is similar to assuming that the debt cost of capital incudes fees. The second adjustment relates to valuation of derivative assets or liabilities on the balance sheet related to interest rate swaps or exchange rate swaps related to debt issues. In the case of interest rate swaps, the valuation of these derivatives reflects fixed interest rates that are higher than prevailing interest rates – a derivative liability – or interest rates in swap contracts being below the prevailing interest rate – a derivative asset. In a sense the valuation of derivative contracts moves the book value of debt on the balance sheet towards the market value. Indeed, if all of debt on the balance sheet had associated interest rate swaps, then adding derivative liabilities and/or deducting derivative liabilities would be an implicit way to reflect market value rather than book value in the bridge between equity value and enterprise value. Unfortunately, of course, some debt issues will probably have fixed rates that are not valued as derivatives. In sum, there are two ways to attempt to reflect the market value of debt in the bridge. The first method is to make a detailed issue by issue market value analysis of debt where the current cost of debt, current credit spreads and current fees are all included in the analysis. If this method is used, then adjustments for the value of debt issuance costs and derivatives should not be made. The second method is to use the book value of debt on the balance sheet and make add or subtract derivatives and debt issuance costs.

A final issue in valuing net debt occurs in an acquisition scenario where the credit quality of a company suddenly changes because of an acquisition or some other re-structuring. If the capital structure is being changed as part of a transaction, the debt should be re-valued to reflect the new level of credit risk. For example assume a company had a liquidity crisis during the financial crisis and had to issue debt at a very high fixed interest rate. If the company that issued the fixed rate debt is purchased by another company that has a strong credit rating, the opportunity cost of debt could decline dramatically. The correct valuation of debt in this case is the value using the new credit spread. In the transaction, the value of debt has increased and the value to the equity holders has declined.

Derivative Assets and Liabilities Associated with Hedging Commodity Prices and Other Items

The discussion above addressed derivative contracts associated with interest rate swaps and exchange rate swaps. Additional derivative assets or liabilities may be generated from commodity price hedges. For example, assume that a company entered into contract to fix the price of oil at very high prices in the middle of 2008 when the price was US$ 147 per barrel. At the end of 2008, the oil price had declined to about US$ 40 per barrel. As the company could realize US$ 147 per barrel from selling oil and buy oil for US$ 40 per barrel, the contract would have had positive value. The question of how value from hedging contracts such as this should be included in valuation of the company depends on how the contract is modeled in the free cash flow calculation. If the hedging contracts are modeled as part of free cash flow, the value of the derivative contracts should not be included in the bridge between equity value and enterprise value. On the other hand, if the hedging contract is not explicitly included in the free cash flow calculation, then the value of the derivative contracts should be included in the bridge.

To demonstrate the principle of valuing derivative contracts, assume that one company signed a favorable contract and another company signed a disadvantageous contract. If the two companies are identical in every other way, the company that signed the advantageous contract should have higher value than the company with the disadvantageous contract. If the contract is modeled as part of free cash flow – for example the company with the advantageous contract realizes higher future revenues – then the increased value is already reflected in free cash flow and enterprise value. On the other hand, if the free cash flow calculation is made at assumed future spot prices, the enterprise value will be the same for both companies. If the value is already included in the enterprise value of the company, the value of the derivative contracts should not be added to enterprise value. If the value of the derivative contracts is not in the enterprise value, the value should be included in the bridge between enterprise value and equity value.

Preferred Stock, Minority Interest and Other Financing Instruments

Preferred stock, minority interest and other instruments used to finance the enterprise should be treated in a similar manner to debt. The fact that the instruments should be valued at market value reflecting the current value of claims on the cash flow should be clear from the discussion of debt. For example, if minority shareholders own 49% of a company, then they should be allocated 49% of the enterprise value no matter what is the minority interest on the balance sheet. Similarly, if a preferred stock instrument has a fixed interest rate plus some kind of equity kicker, the preferred stock should be measured at market value that depends on the risk and the prospective cash flow of the instrument. As with debt and assets discussed above, the minority interest and the preferred stock should be included in the WACC calculation. For example if minority interest represents 49% of the company, it should be included at market value in the WACC just like the debt. Assume a second company has no minority equity and the same enterprise value, implying that the amount of debt is higher. The example demonstrates that company with the minority interest should have lower volatility and lower cost of equity capital because of the relatively lower amount of debt. To derive the appropriate cost of capital that should be used in measuring the value of free cash flow, the minority interest should be included in the WACC calculation which effectively increases the equity percentage and lowers the cost of capital.

As in the case of debt, in practical situations valuing the minority interest may be complex because of the lack of market data. Alternatives include applying a P/E ratio to the income attributed to minority interest or to use the ratio of minority interest to common equity in establishing value. To apply the latter method, one can compute total value of minority and common equity and then split the equity into the common equity portion and the minority equity portion.

Current Liabilities and Long-term Payables

The treatment of accounts payable and other amounts owed to suppliers of labor and materials is analogous to the treatment of accounts receivable. One can make the argument that accounts payable are not much unlike other debt obligations. Instead of paying interest, the cost of operating expenses is somewhat higher than the expenses would otherwise be. It would theoretically be possible to separate the operating expenses into a component that represents interest expense and pure expenses. If the interest was excluded from enterprise value, then the present value of free cash flows would be higher and the level of accounts payable should be included in the bridge between equity value and enterprise value. In fact, this would be practically impossible, and the implicit financing costs of the accounts payable cannot be separated and are included in free cash flow. The difference between long-term accounts payable and short-term payable is not important in evaluating the bridge between equity value and enterprise value. The key question is whether the operating expenses related to the accounts payable are included in the free cash flow. If a supplier offers very attractive payment terms, but the cost of supplying materials is increased, the treatment depends on whether the increased operating expenses is included in cash flow.

In addition to the financing cost, the treatment of payables is affected by the question of whether the level of accounts payable changes over time. In general, the accounts payable are assumed to increase with the growth of the company implying that the accounts payable provide cash flow and are never a liability that must be repaid. If the accounts payable continue to grow and the growth is included in the working capital changes in the free cash flow, then there is no cash flow that must be incurred by equity investors that is not already included in free cash flow (as changes in working capital). Indeed, the accounts payable never really come due as a cash obligation to equity investors. Instead, they are replaced by new payables and the implicit financing cost is included in the operating expenses. If there is a long-term payable that will come due and not be replaced, then the treatment depends on how the repayment of the payable is included in the free cash flow. If repayment of the payable is included as a negative item in the free cash flow, then no adjustment in the bridge is necessary. However, if the long-term payable will not be replaced and it is not included in the cash flow, then the long-term payable should be both included in the bridge and in the WACC calculation. In the WACC table, the long-term accounts payable should have no interest cost as long as the implied cost of capital is correctly included in the operating expense and free cash flow.

Unfunded Pension Liabilities

Unfunded liabilities on the balance sheet arise because of defined benefit pension plans that companies provide to their employees where the amount of benefit is defined and does not vary as a function of financial market returns. The amount of money that must be paid in future is computed at its present value and compared to amounts that are held in a trust fund to pay the future expenses. If the amount in the pension trust fund is below the present value that must be paid to employees, then a liability is recorded on the balance sheet. Unfunded pension obligations are liabilities that must eventually be repaid by shareholders and the cash outflow associated with putting money into the trust funds is generally not included as a cash outflow in the free cash flow calculation. The company could borrow money to fully fund the pension trust fund and eliminate the liability implying that the unfunded liability should be treated as debt in both the bridge between equity value and enterprise value and in the WACC calculation. To illustrate this, consider an example of two identical companies in every respect except that one company funds its pension plan while carrying debt on its balance sheet while the other company does not fund the pension plant. For these two companies, both the enterprise value and the equity value should be the same as one company has simply chosen a different financial strategy. If the cash flow declines, then debt leverage aggravates the decline from the standpoint of equity owners implying the cost of equity is higher with leverage. In the same way, if a company has an unfunded pension plan, that fixed obligation magnifies the effect of reductions in cash flow to equity owners in the same way as fixed obligations from debt.

As with other issues, where an item is included in the bridge between equity capital and enterprise value, the item should also be included in the WACC calculation. If a company is not fully funding its pension plan, one could think of the company borrowing from the pension trust fund. There is no economic difference between borrowing money from a bank and borrowing money from the trust fund. In theory, the discount rate used in valuing future obligations should be relatively low reflecting the fixed nature of the obligations. A relatively low discount rate also increases the pension liability on the balance sheet. If a low discount rate is used, the pension obligations should also be included as a component of the WACC at that discount rate. If a high discount rate would be used, the liability would be undervalued. To offset the undervaluation of the pension obligations, the higher discount rate could be used in the WACC calculation.

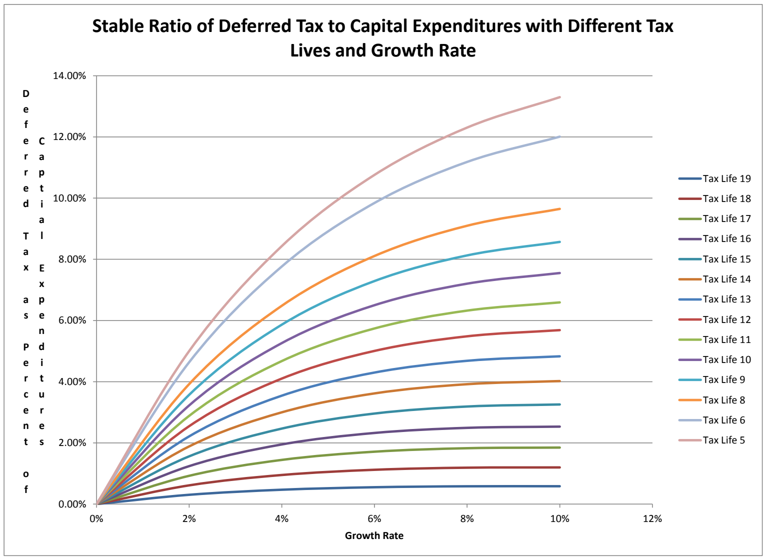
Accumulated Deferred Taxes

Deferred taxes may be the most complex and important item to evaluate in considering the difference between enterprise value and equity value. Deferred taxes are generally recorded as a liability on the balance sheet. If deferred taxes really cause equity investors to incur investors to incur cash outflows that are not included in the free cash flow, then the value of the liability should be deducted from enterprise value. However, if the company continues to grow and new make capital expenditures, then the deferred tax liability is continually replaced with cash flow from reduced taxes associated with new depreciation that creates additional deferred taxes. From this perspective, deferred taxes continually increase cash flow and add to the value of the firm. The discussion below demonstrates that the appropriate treatment of deferred tax is to directly include the changes in deferred tax as a part of the free cash flow. To see why deferred taxes should be directly incorporated in cash flow, it is useful to briefly work through the accounting for deferred taxes and the treatment of taxes in the free cash flow. After the accounting is understood, the treatment in valuation is addressed. In discussing the valuation, two options are considered. The first option is including deferred taxes as part of the bridge between equity value and enterprise value. The second option is including the deferred tax directly as a part of the free cash flow and excluding the deferred tax from calculation of the bridge between equity value and enterprise value.

In computing book income taxes, book depreciation and intangible amortization is assumed to be used in the calculation. Further, if the book pre-tax income is negative, then negative taxes are recorded on the income statement. Similarly, the tax calculation in the free cash flow is based on the book depreciation and amortization as it is computed as EBIT multiplied by the tax rate. Actual taxes paid by a company are a function of tax depreciation and allowable amortization allowable for tax purposes as well as other differences between calculation of actual taxes and calculation of book income. The most common example of an item that creates deferred tax is a faster write-off of depreciation for tax purposes than for tax purposes. In early years of the life of an investment, the tax depreciation is higher than book depreciation which means that taxes paid are lower than taxes booked. Without considering construction of other new assets, the tax depreciation is below book depreciation and taxes paid are higher than taxes booked. The nominal amount of taxes that must be paid in the future due to the use of faster depreciation is measured by the amount of accumulated deferred taxes on the balance sheet. The mechanics of deferred tax accounting for a single asset is illustrated below. Note in computing the change in deferred tax that is added to cash flow, the tax depreciation expense is subtracted from the book depreciation expense.

Assumptions					
Book Life	4				
Tax Life	2				
Tax Rate	40%				
Capital Expenditures	100,000				
Book and Tax Depreciation					
Year		1	2	3	4
Book Depreciation		25,000.00	25,000.00	25,000.00	25,000.00
Tax Depreciation		50,000.00	50,000.00	-	-
Tax - Book Depreciation		25,000.00	25,000.00	(25,000.00)	(25,000.00)
(Tax - Book) x Tax Rate		10,000.00	10,000.00	(10,000.00)	(10,000.00)
Accumulated Deferred Taxes					
Opening Balance		-	10,000.00	20,000.00	10,000.00
Add: Deferred Tax Change		10,000.00	10,000.00	(10,000.00)	(10,000.00)
Closing Balance	-	10,000.00	20,000.00	10,000.00	-

The problem with the example above is that as capital expenditures are made, tax depreciation keeps increasing and is continually more than book depreciation. Indeed, after a cycle of plant in which after retirements begin, the changes in deferred tax stabilize to a fixed level relative to capital expenditures. Further, the level of accumulated expenditures stabilizes relative to net plant. As capital expenditures continue, the deferred tax liability never comes due and the deferred tax liability continues to increase. The stable level of deferred tax relative to the difference between book and tax depreciation lives is illustrated in the graph below with different growth rates in capital expenditures. The graph assumes that deprecation is computed on a straight line basis.



In computing cash flow, one could include the change in deferred tax directly in the free cash flow. Through including the deferred tax as an addition to cash flow, taxes in the free cash flow calculation reflect actual taxes paid. Further, when computing the terminal value in the stable period, the stable ratio of deferred tax to capital expenditure can be used. Through modeling deferred taxes directly in the cash flow including the terminal value, the deferred taxes are added to the enterprise value rather than being subtracted as part of the bridge between enterprise value and equity value. It would be incorrect to reduce the enterprise value that is already understated because it does not incorporate the on-going benefit of tax depreciation expense relative to book depreciation expense.

Deferred taxes arise for many other reasons than the difference between book and tax depreciation. Other examples include net operating loss carry forwards and deferred taxes arising from an acquisition where a tax free exchange is used for tax purposes. The reason deferred taxes arise from an acquisition is because write-ups in plant and intangible assets increase depreciation and amortization for book purposes without affecting tax depreciation. This increases the deferred tax liability because tax depreciation is less than book depreciation suggesting an increased liability exists. As intangible assets are amortized and increased plant balances are written-off the deferred tax change can be a negative number. In reflecting the effects of an acquisition in discounted cash flow as well as net operating loss, the change in deferred tax should be carefully modeled as part of cash flow. Similarly, if deferred taxes arise from a net operating loss carry forward (creating a deferred tax asset rather than a liability) then the deferred tax should also be included in free cash flow.

Stock Options

In making a valuation using the discounted cash flow model, the objective of valuation is measurement of the value per share to common shareholders. If stock options have been granted to management as part of their compensation package, exercise of the options increases the number of shares without providing additional capital. These stock options can spread the equity and enterprise value over more shares and reduce the value per share to existing common shareholders. To illustrate the treatment of stock options, consider a hypothetical example of two companies that are identical in every way, except that one company pays its executives with cash and another company pays its employees through stock options. The difference in method of payment should not affect the value to common shareholders. The company that pays its executives which stock options has more cash because of not paying its employees, but it also has more cash and a higher equity balance. The stock options are accounted for as operating expenses implying that the EBITDA and the free cash flow should be the same for both of the companies. However, the equity value of the company with stock options appears to be higher because of the increased cash (or decreased debt) on its balance sheet.

To adjust for the value of stock options, two approaches can be adopted as illustrated on the table below. The first approach is to divide the equity value by the diluted shares rather than the basic shares. The second approach is to quantify the value of stock options and include the value of the stock options as a deduction when adjusting the enterprise value. When using the second approach, the equity value should be divided by the basic shares rather the diluted shares. Details of stock options in terms of the number of options, exercise prices and expiration dates are often presented in notes to financial statements. With this information it is quite easy to compute the value of stock options using different volatility, stock price and interest rate assumptions.

Valation of Stock Options

	Pay Executives with Cash	Pay Executives with Stock
Profit and Loss, Free Cash and EV		
Revenues	1,000	1,000
Operating Expense - Cash	400	0
Operating Expense - Stock Options	0	400
EBITDA	600	600
WACC	10%	10%
Enterprise Value assuming Constant	6,000.00	6,000.00
Balance Sheet		
Cash	100.00	500.00
Other Assets	5,000.00	5,000.00
Total	5,100.00	5,500.00
Equity	5,100.00	5,500.00
Valuation with Diluted Shares		
Value of Common Equity (Incl Cash)	6,100.00	6,500.00
Common Shares - Basic	100.00	100.00
Value per Share	61.00	65.00
Added Shares	-	6.56
Diluted Shares	100.00	106.56
Value per Diluted Share	61.00	61.00
Valuation using Stock Grant Value in Bridge		
Value of Stock Grants	-	400.00
Enterprise Value + Cash - Stock Grants	6,100.00	6,100.00
Adjusted Value per Basic Share	61.00	61.00

Accumulated Other Comprehensive Income

Accumulated other comprehensive income arises because some assets on the balance sheet such as derivative liabilities are not recorded on the income statement. Other items are related to currency translation gains or losses whereby financial assets are translated to a different currency. No valuation is generally required for accumulated other comprehensive income as items such as derivative assets have already been discussed. The foreign translation adjustments should also already be reflected in the market value of debt and equity.

Summary

Given the length of this section, a summary of the different items is presented below.

Summary of Treatment of Balance Sheet Items in the Bridge Between Enteprise Value and Equity Value

Balance Sheet Item	General Treatement in Bridge	Theoretical Treatment in WACC	Comments and Difficulties
Surplus Cash	Include as Negative	Include in WACC	WACC treatment depends on Cost of Equity Calculation
Operating Cash	Exclude	Exclude	Could argue that operating cash has same treatment as surplus cash
Short-term Accounts Recievable and Inventory	Exclude	Exclude	Implicit financing income from A/R in EBITDA
Long-term Notes Receivable and Accounts Receviable	Include if One-Time Event	Include as Negative	Exclude if redemption is included in free cash flow
Deferred Expenses and Prepaid Expenses	Exclude if Expense Reflects Cash	Exclude	Include if operating expense is not reduced to reflect cash
Assets not currently producing cash flow	Include at Market Value	Adjust WACC	Exclude if effect of assets in capital expenditures or in assets sales
Non Assocaited Investments	Include if income not in EBITDA	Adjust WACC	If non-associated income is in EBITDA, exclude from bridge
Discountinued Assets and Liabilities	Include if assets sales not in Cash Flow	Adjust WACC	If asset sales included in free cash flow, exclude from bridge
Debt	Include at Market Value	Include in WACC	Could be at book value if include derivative liabilities
Unfunded Pension Obligations	Include at Balance Sheet Value	Adjust at Discount Rate	Liabilities should be valued similar to debt interest rate
Long-term payables	Similar to Long-term Notes Receivable	Adjust WACC	Depends if the payables are one-off and included in cash flow
Accuumlated Reclamation Costs	Include if No Associated Investments	Adjust WACC	Unless cash flow includes outflows or trust fund investments
Derivative liabilities (assets) related to interest rate swaps	Exclude if Debt at Market Value	Exclude	Could include if debt is valued at book
Derivative liabilities (assets) related to foreing exchange swaps	Exclude if Debt at Market Value	Exclude	Could include if debt is valued at book
Derivative liabilities (assets) related to commodity hedges	Include if Not in Cash Flow	Adjust WACC	If contracts modelled in cash flow, exclude from bridge
Preferred Stock	Same Treatment as Debt	Include in WACC	Market value depends on assumed cost of capital
Minority Interest	Include at Market Value	Include in WACC	Can compute proportion of value owned by minority interest
Accumulated Deferred Taxes	Exclude and Model Changes in Cash Flow	Exclude	Should compute statble level of deferred taxes in terminal value
Stock Options	Exclude and Divide by Diluted Shares	Adjust WACC	Can directly compute value of stock options and include in bridge

Case 2: Adjustments in Computing EV/EBITDA Ratio

When computing the EV/EBITDA ratio for a company, calculation of the enterprise value implies that the bridge between equity value and enterprise value must be included as the equity value is known but the enterprise value must be derived. For example, in computing the implied EV/EBITDA ratio in an acquisition, one would typically begin with the consideration in the acquisition or the amount paid for the equity of the company being purchased. To arrive at enterprise value from the equity value, the same sort of adjustments should be made as those described above for computing the bridge between discounted cash flow are required.

From a theoretical perspective, the adjustments in computing the equity value from the enterprise value discussed above are no different from beginning with the market capitalization. However, many of the issues in considering the bridge were ambiguous and depend on the computation of EBITDA and free cash flow. For example, inclusion of the value of non-associated assets in the bridge should occur if income from non-associated assets is not included in EBITDA. The problem with making all of the adjustments when computing the EV/EBITDA ratio is that the reason for computing the ratio in the first place is to make comparisons with other companies and/or other transactions. If the enterprise value is computed in a very precise manner considering all of the adjustments discussed above, but the comparison sample is computed in a different manner, the valuation will be distorted. Consider a highly contrived example below where the difference between EV/EBITDA with and without unfunded pension plans is the same for the company as for a comparative sample. In this extreme case, it is better to compute the EV/EBITDA for the company in a consistent manner as for the comparative sample. If the company in question has a different relative amount of unfunded liabilities than the comparative average, then valuation biases will exit.

	EV/EBITDA with Pension Adjustment	EV/EBITDA without Pension Adjustment
Compariosn Companies		
Company A	8.8	8.5
Company B	8.5	8.3
Company C	9.4	9.2
Company D	8.4	8.1
Median	8.65	8.4
Company in Question		
Market Capitalization	3,500	3,500
Net Debt	2,000	2,000
Unfunded Pensions	500	0
Total Enterprise Value	6,000	5,500
EBITDA	694	694
EV/EBITDA with pensions	8.65	8.65
EV/EBITDA w/o pensions	7.93	7.93
Comparison EV/EBITDA	8.65	8.40
Implied Enterprise Value	6,003	5,830
Implied Equity Value	3,503	3,830

In making comparative samples, it would be best if the EV/EBITDA ratio were calculated by carefully including each item at the appropriate market value. If this is not possible, it is essential to understand how the ratio is computed in the sample.

Case 3: Adjustments in Return on Invested Capital

When computing the return on invested capital, one should divide the net operating income less adjusted taxes (NOPLAT) by the invested capital. The NOPLAT can be computed as the EBIT multiplied by one minus the tax rate (generally the statutory tax rate). However calculation of invested capital in the denominator raises is not as straightforward and raises similar questions as the bridge between enterprise value and equity value. The invested capital should be measured at the historic amount of capital invested rather than the market value.

Invested Capital

Invested capital can be computed from two perspectives. The first is the perspective of capital provided to generate EBITDA. Note that some capital is provided to fund non-associated assets that generate income that is not included in EBITDA. Similarly cash on the balance sheet generated interest income that is not included in EBITDA. Anything that provides cash flow that is below the EBITDA line should not be included in EBITDA. The second perspective is that of including net assets that do generate EBITDA. These assets include current assets other than cash, goodwill, associated assets and anything else that is necessary to run the operating part of a business.

Note how can move from bottom section to top section if the item does not produce EBITDA but effects value. Examples of project assets that may be not be in EBITDA in projections.

	2004	2005	2006	2007	2008	2009	2010
Invested Capital -- Capital Provided Perspective							
Total Equity	22,641	38,394	461,666	1,097,267	1,513,042	2,652,787	3,454,945
Total Debt	13,700	48,723	80,697	108,165	198,470	174,958	237,391
Less: Surplus Cash (providing interest income not in EBITDA)	(3,771)	(18,300)	(316,639)	(651,645)	(847,319)	(1,150,837)	(1,199,852)
Less: Other Assets (providing income not in EBITDA)	(906)	(286)	(2,896)	-	-	(48,217)	(43,488)
Total Invested Capital	31,664	68,531	222,828	553,787	864,193	1,628,691	2,448,996
Invested Capital -- Perspective of Assets and Liabilities Required to Generate EBITDA							
Current Assets Less Cash	7,811	9,520	52,592	131,823	289,258	565,471	693,222
Less: Current Liabilities Less Debt	-5,424	-13,771	-32,423	-141,987	-346,826	-366,368	-443,090
Net Plant Assets	29,277	73,778	178,868	430,104	842,622	988,782	1,430,789
Project Assets	0	0	27,515	87,688	61,993	132,496	320,140
Deferred Tax Assets	0	0	0	36,603	39,481	152,194	259,624
Investment in Related Property	0	0	0	33,449	33,829	25,000	0
Goodwill	0	0	0	0	0	286,515	433,288
Less: Accrued Liability	0	-917	-3,724	-13,079	-35,238	-92,799	-132,951
Less: Other Liabilities	0	-79	0	-10,814	-20,926	-62,600	-112,026
Total Invested Capital	31,664	68,531	222,828	553,787	864,193	1,628,691	2,448,996
Difference	0	0	0	0	0	0	0

In computing the return on equity capital, the idea of the ratio is to measure the current amount of income provided to equity holders divided by the amount they invested directly or indirectly through not retrieving dividends on the income that has been earned. With the exception of accumulated other comprehensive income, computing the equity value on the balance sheet is straightforward, but evaluating the invested capital raises a number of questions. A second difference in computing invested capital from equity capital relative to computing equity value from enterprise value is that historic EBITDA is used rather than forecasted free cash flow. This means that items in the bridge cannot be directly associated with different calculation techniques in free cash flow.

As with the EV/EBITDA ratio, the approach to computing the invested capital depends on how the return on invested capital is used. If the return for one company is compared to other companies, it is more important to be consistent in computing the ratio than to spend a lot of time on each detail. Other applications of the return on invested capital include comparing the return with the WACC and evaluating trends in the return in financial models. For these applications, the comparison with other companies is not part of the calculus and the adjustments discussed below should be considered.

Before delving into the adjustments for enterprise value, the treatment of accumulated other comprehensive income in the equity investment must be addressed. Accumulated other comprehensive income appears on the balance sheet because, among other reasons, derivative assets and liabilities as well as currency translation adjustments are placed on the balance sheet without affecting income. If an activity is considered hedging but causes the value of derivatives to deviate in market value, the market value of the derivative is included on the balance sheet, but the income associated with the change in value is not included in income. If an asset or liability has a change in value, it generally affects income. However, because some derivative assets and liabilities do not affect income, accumulated other comprehensive income is used instead. Because this account arises from adjustments to asset accounts, the amount of accumulated other comprehensive income should not directly or indirectly represent investments made by equity investors. Therefore, when computing the return on equity capital, the denominator should be adjusted by deducting the amount of accumulated other comprehensive income.

Once equity investment is established, the amount of invested capital that is deployed to generate the EBITDA or the NOPLAT must be derived. For example, the debt investment made by lenders was made in part to generate NOPLAT implying that it should be included in invested capital. On the other hand, investments made surplus cash and other investments that are not generating EBITDA or NOPLAT, but are generating interest income should be deducted from the equity capital and the debt capital that has been raised to finance the company. Investments in land and discontinued assets that are not producing EBITDA should be deducted from the invested capital at the investment cost because debt and equity raised to finance these items does not generate EBITDA. Thus, the general principles in making adjustments that move from equity capital to invested capital are similar to the enterprise value versus equity value adjustments described above. In the case of non-associated investments, if the income from non-associated investments is included in EBITDA, then the investment made to generate this income should not be segregated from the debt and equity investment made to generate the EBITDA. On the other hand, if income from non-associated assets is not included in EBITDA, then the investment made in the non-associated assets should be deducted from the debt and equity funding that was made to finance the company.

On the liability side, un-funded pension obligations should be considered as debt for the same reasons as discussed above for the bridge between enterprise value and equity value in the discounted cash flow model. Consider two companies, one with a surplus in its pension trust fund and another with a liability for unfunded pension obligations. For the company with the pension surplus, part of the debt and equity investment in the company made by investors was implicitly to finance the extra amount in the pension fund. As the investment attributed to the surplus pension fund does not generate EBITDA, it should not be include in the invested capital associated with NOPLAT. Where the company has a deficit in the pension plan, the principle is the same. Here, not enough debt and equity have been invested to support the NOPLAT and the EBITDA meaning that the invested capital must be increased.

In the case of deferred taxes, an adjustment to invested capital should not be made. This is because the deferred taxes that can in part finance an investment are not contributed by investors. To illustrate issues associated with deferred tax, assume that two companies are identical in every respect except that one company is able to use accelerated depreciation while the second is not. The company that uses deferred taxes should have higher value and a higher return on invested capital. Further, one can argue that the change in value is the result of operating activities rather than financing activities. If deferred taxes are included in the denominator of the return on invested capital calculation, then the two companies will have the same return on invested capital which will not reflect the advantageous position of the company that can use accelerated depreciation. If accumulated deferred taxes are not included in the invested capital calculation, the company that can use accelerated depreciation and has financed its operations with lower capital will have hider return on invested capital as should be the case. As with many of the items, a good way to think about the treatment is to develop a simple example.

Derivative liabilities and assets do not represent investments made by debt and equity investors and should not be part of the adjustment to derive invested capital from equity capital. Consider two companies, one with an interest swap that is significantly in the money and creates a derivative asset while the other company has no derivatives. The company with the interest rate swap has higher value, but that value is due to the change in debt value. While an argument could be made to reduce the invested capital by the derivative assets and thereby reflect the higher value as higher return on invested capital the concept of return on invested capital should not be influenced by changes in financing costs. If the value of derivatives is included in the return on invested capital, similar ideas could be used for the change in value other debt and equity investments. For example, if the cost of equity changes because the risk free rate changes, one could attempt to incorporate the change in value in the return on invested capital calculation. The change in the value of a derivative reflects future expectations while the objective of computing the return on invested capital is to evaluate the current income of a company relative to the amount of money that is invested in the company.

Other equity items such as minority interest and preferred stock should be included at the invested amount as with debt and common equity because these amounts represent capital that has been invested that demands a return. If vendor capital has been used and is booked as a long-term payable or deferred debit, and if the implied interest cost is not in EBITDA, then it also could be counted as invested capital. Unlike the case for enterprise value and equity value, the value of stock options should not be considered in the invested capital calculation. If executives are paid through stock options, the EBITDA is already reduced and the adjustment to equity capital reflects implied capital that has been invested in the company.

A summary of adjustments that should be made in moving from common equity investment reported on the balance sheet to invested capital for purposes of computing the return on invested capital is listed on the table below.

Summary of Treatment of Balance Sheet Items in the Bridge Shareholders Equity and Invested Capital

Balance Sheet Item	General Treatement in Bridge	Comments
Accumulated Other Comprehensive Income	Subtract from Equity Capital	AOCI does not represent capital invested
Long and Short term Debt and Financial Leases	Include as Invested Capital at Book Value	Debt and equity capital are invested to support EBITDA
Preferred Stock	Same Treatment as Debt	Preferred stock can be invested as well as debt
Minority Interest	Same Treatment as Debt	Minority interest is similar to common equity
Surplus Cash	Subtract from Debt and Equity	Some invested capital dedicated to surplus cash
Operating Cash	No Adjustment	Operating cash investment to support EBITDA
Short-term Accounts Recievable and Inventory	No Adjustment	A/R and Inventory investment to support EBITDA
Long-term Notes Receivable and Accounts Receviable	Subtract from Debt and Equity	Investment does not generate EBITDA
Deferred Expenses and Prepaid Expenses	No Adjustment	Same rationale as A/R
Assets not currently producing cash flow (land etc.)	Subtract from Debt and Equity	Investment made to support assets not necessary for EBTIDA
Non Assocaited Investments	Subtract if Income not in EBITDA	If non-associated income is in EBITDA, subtract from invested capital
Discountinued Assets and Liabilities	Subtract if Income not in EBITDA	Some invested capital dedicated to discontinued operations
Unfunded Pension Obligations	Add to invested capital	Invested capital would have to be used to fund pensions
Derivative liabilities (assets) related to interest rate swaps	No Adjustment	Does not represent invested capital
Derivative liabilities (assets) related to commodity hedges	No Adjustment	Does not represent invested capital
Accumulated Deferred Taxes	No Adjustment	Not capital invested to support EBITDA
Stock Options	No Adjustment	Capital invested included in equity

Dividend Discount Model and P/E Ratio to Derive Cost of Capital

The dividend discount model can be used to estimate the cost of capital given the value of a stock along with an estimate of the growth rate in dividends. The model estimates the cash flow that will accrue to investors from owning stock and then discount that stream of money to ascertain its value today. By assuming that marginal investors, who are the ones buying and selling shares, believe the growth rate in dividends is constant forever, one can establish the well known dividend discount equation for computing the cost of capital. The value of a share is the next anticipated dividend divided by the difference in the cost of equity and the growth rate in dividends (the mathematics of the formula requires using the next year cash flow rather than the current period dividend):

$$P_0 = D_1/(k-g)$$

Rearranging, the formula becomes:

$$k = D_1/P_0 + g.$$

This equation implies that lower growth results in a lower estimated cost of equity capital as a lower discount rate must be used to arrive at the same share price. Because the model estimates the cost of capital from assumed investor valuation models, the inputs to the model -- dividend yield and the growth

rate -- are not the direct drivers of risk and cost of capital as was the beta and the equity risk premium in the CAPM.

The model has many problems including the fact that long-term growth rates in dividends predicted by sell side analysts and often used in the model are generally optimistic; that earnings growth rates cannot be substituted for dividend growth rates when dividend payout ratios are changing; that assuming a company will stabilize after a certain time period presents subjective estimates as to when will stabilization occur and what will be the long-term growth rate and whether the model should be applied using a sustainable growth rate.

Inference of cost of capital from an investor valuation model does not require use of dividend forecasts to establish stock price valuation. Other valuation formulas can be used to infer the cost of equity capital, instead of using the present value of projected dividends as the underlying valuation model. In particular, a model that is used to evaluate price to earnings ratios can be the basis for estimating the implicit cost of equity capital. A formula that explains the P/E ratio in terms key value drivers – the ability to earn more than the cost of capital and the ability to grow that difference – is given by the equation:

$$P_0/E_1 = [1-g/ROE]/[k-g]$$

In this formula, P/E is the forward price to earnings ratio, g is the growth rate in earnings per share, ROE is the earned return on equity and k is the cost of equity capital. Through re-arranging the P/E formula, one can compute the cost of equity from the P/E ratio. In this formula, the cost of equity capital is driven by the growth, the return on equity, and the forward earnings per share:

$$k = [EPS_1 \times (1-ROE/g)]/P_0 + g$$

If you want to apply either the P/E or the standard dividend discount model, you can use a file provided on the CD. After you enter ticker symbols, the file retrieves data from the Yahoo.Finance website and then computes the cost of equity using one of the two formulas discussed above. Growth rates come either from investment analyst projected five year growth rates which are often used in the analysis.

Growth rates can be extracted from one year projected growth rates or from the sustainable growth rate formula:

$$\text{Sustainable growth} = ROE \times (1-\text{dividend payout})$$

	Laclede									
	Select Method ---------------------------------->			Forward	Five Year Projected	Reported				
			Ticker	PE Ratio From Yahoo or Forward EPS/Share Price	Growth Over Next 5 Years (Growth Page from Yahoo)	Reported by Yahoo (Base Page)	**Equity Cost of Capital from PE Formula**	**Forward Dividend Yield**	**Equity Cost of Capital from DCF Model**	
	Company Name		Ticker	PE Ratio	Growth	ROE	**Equity Cost**	Dividend Yield	Equity Cost	Mkt/Book
1	Microsoft		MSFT	9.82	10.2%	50.0%	**18.32%**	2.77%	12.99%	4.97
2	Nokia		NOK	11.15	13.5%	27.5%	**18.07%**	3.95%	17.45%	N/A
3	Vodaphone		VOD	8.13	6.7%	7.5%	**7.95%**	4.10%	10.80%	0.89
4	General Electric		GE	11.40	8.7%	16.4%	**12.81%**	11.33%	20.00%	1.08
5	Google		GOOG	15.34	19.0%	16.6%	**18.06%**	0.00%	19.00%	4.05
6	Fed Exp		FDX	14.76	5.8%	4.9%	**4.61%**	0.88%	6.67%	0.98
7	AES Corp		AES	5.50	14.2%	35.6%	**25.13%**	0.00%	14.20%	1.13
8	Boeing		BA	7.45	8.4%	68.9%	**20.19%**	4.46%	12.86%	N/A
9	Freeport McMoran		FCX	15.63	6.0%	1.0%	**-25.99%**	0.00%	6.00%	7.73
10	IBM Corp		IBM	10.43	9.8%	58.8%	**17.82%**	1.96%	11.79%	10.03
11	McDonalds		MCD	13.58	9.0%	30.1%	**14.18%**	3.53%	12.56%	4.70
12	Sprint Nextel		S	(19.05)	3.4%	1.0%	**15.75%**	0.00%	3.36%	0.62
13	Southwest Air		LUV	10.09	13.3%	3.0%	**-20.94%**	0.29%	13.62%	1.05
14	UPS		UPS	15.98	10.2%	31.7%	**14.43%**	3.41%	13.60%	7.71
15	Con Ed		ED	11.65	2.5%	9.8%	**8.90%**	6.01%	8.55%	1.11
16	BP		BP	7.49	4.5%	22.9%	**15.22%**	8.13%	12.63%	1.42
17	Exxon Mobil		XOM	11.27	8.3%	38.5%	**15.28%**	2.27%	10.60%	3.09
18	Daimler Chrysler		DAI	12.50	5.0%	4.8%	**4.70%**	1.87%	6.87%	N/A
19	Deutche Telcom		DT	11.39	4.3%	3.6%	**2.62%**	9.72%	14.02%	N/A
20	Goldman Sachs		GS	15.04	13.5%	4.3%	**-0.58%**	1.57%	15.07%	1.03
21	Walmart		WMT	13.90	11.6%	20.4%	**14.71%**	2.03%	13.63%	3.22
22	Yahoo		YHOO	31.02	15.5%	4.1%	**6.48%**	0.00%	15.53%	1.60
23	Nigas		GAS	11.80	4.5%	12.5%	**9.91%**	5.55%	10.05%	1.56
24	Laclede		LG	15.24	3.5%	14.2%	**8.44%**	3.95%	7.45%	1.70

By far the most difficult part of the dividend discount model or the P/E equation is estimating the future growth rate in earnings or dividends. A number of financial economists have demonstrated that investment analyst estimates of growth in earnings, particularly the five year forecasts are strongly biased in an upward direction. If the stock market is anywhere near efficient and analyst growth rates are consistently high, stock prices should reflect the unbiased growth estimates rather than the upwardly biased sell-side analyst forecasts. This means that use of analyst forecasts as the growth rates in the dividend discount model or the P/E formula will over-estimate the cost of capital inferred from the DCF valuation equation.

Some of the economists who have noted the bias in growth rates are summarized below. For example, their text titled "Valuation: Measuring and Managing the Value of Companies" published in 2005, Tim Koller, Marc Goedhart and David Wessels state that "*analyst forecasts focus on the short term and are severely upward biased*."[3] Enrique Arzac comments on the difficulty of predicting growth rates and the potential for the DCF model to over-estimate the cost of equity as follows: "*The problem with [the dividend discount] approach is that long-term dividend growth rate of an individual company cannot be estimated with any degree of precision. Hence, the dividend growth model is not likely to produce reliable estimates of the cost of equity capital of individual companies….A number of empirical studies have documented optimistic bias in analysts' opinions….*"[4] Claus and Thomas conclude that earnings and dividend growth rates used for the DCF model "exhibit substantial optimism bias and need to be adjusted downward."[5] Louis Chan and his coauthors conclude that "*over the period 1982 to 1998, the median of the distribution of IBES growth forecasts is about 14.5 percent, a far cry from the median realized five year growth rate of about 9 percent for income before extraordinary items.*"[6] Fama and French state that "*In short, we find no evidence to support a forecast of strong future dividend or earnings growth at the end of our sample period*."[7]

Questions of whether analyst forecast of earnings provide any predictive power have been studied for many years. In one of his textbooks, Aswath Damodaran finds that any superiority of analyst forecasts deteriorates quickly as the forecast period is extended.[8]

> *The analyst forecasts outperform the time series model for one-quarter ahead and two-quarter ahead forecasts, do as well as the time series model for three-quarter ahead forecasts and do worse than the time series model for four-quarter ahead forecasts. Thus, the advantage gained by analysts from firm-specific information seems to deteriorate as the time horizon for forecasting is extended. There is an intuitive basis for arguing that analyst predictions of growth rates must be better than time-series or other historical-data based models simply because they use more information. The evidence indicates, however, that this superiority in forecasting is surprisingly small for long-term forecasts and that past growth rates play a significant role in determining analyst forecasts. . . . Analyst forecasts may be useful in coming up with a predicted growth rate for a firm but there is a danger to blindly following consensus forecasts. Analysts often make significant errors in forecasting earnings, partly because they depend upon the same data sources (which might have been erroneous or misleading) and partly because they sometimes overlook significant shifts in the fundamental characteristics of the firm. The secret to successful valuation often lies in discovering inconsistencies between analysts' forecasts of growth and a firm's fundamentals.*

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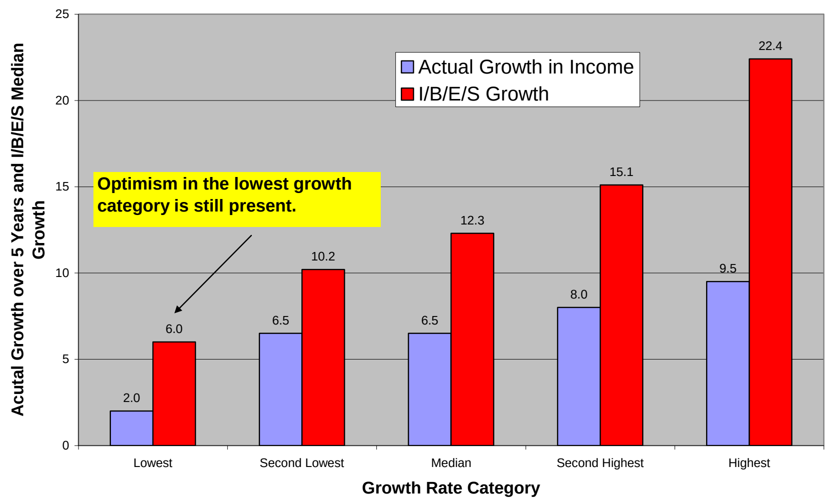
[3] Koller, T., Goedhart, M., Wessells, D., 2005, Valuation Measuring and Managing the Value of Companies, Hoboken, New Jersey, John Wiley & Sons p. 305.
[4] Ibid, Arzac, Enrique, 2005, pp. 43-44.
[5] Ibid, Claus and Thomas, 2001, p. 1662.
[6] Chan, L., Karceski, j and Lakonishok, J, 2003, The Level and Persistence of Growth Rates, Journal of Finance, 58, p. 672.
[7] Fama and French, 2002.
[8] Damarodan on Valuation: Security Analysis for Investment and Corporate Finance, pp 165-166.

The article by Louis Chan allows comparison of growth rates made by analysts that are compiled by a service called I/B/E/S. The graph categorizes growth rates for companies with different levels of growth and demonstrates that even companies with low growth have strongly upward biased projections. For each category, the bar that shows actual growth is far below the projected growth over the same period.

IBES Growth and Actual Growth from Chan Article



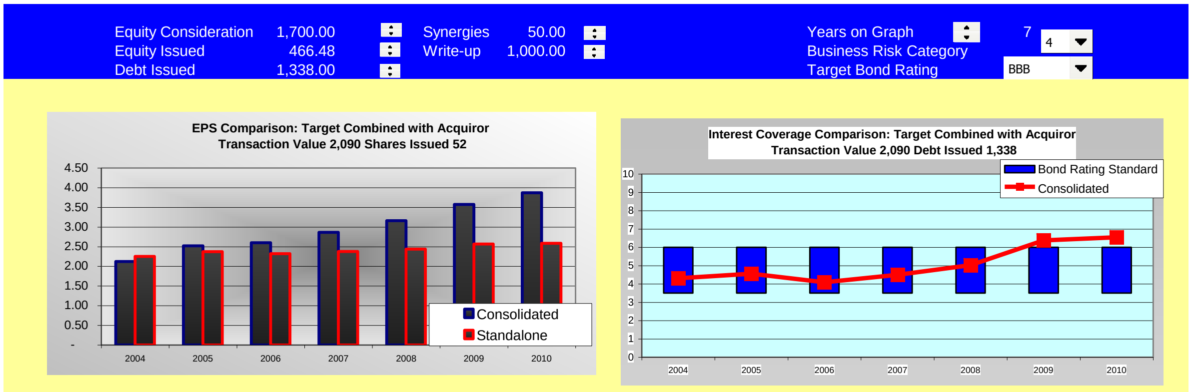
Valuation in Alternative Models

All four model types introduced in this chapter – corporate models, project finance models, leveraged acquisition models and integrated merger models -- compute financial statements for the company being evaluated. However, the manner in which valuation is derived from the models and the ways risks can be measured is very different. Corporate models generally compute value from projected earnings or from forecasted free cash flow using the discounted cash flow ("DCF") method. The foundation of this approach is the presumption that the value of real assets is not dependent on the manner in which the assets are financed. A simple definition of free cash flow – cash flow that accrues to debt investors, equity investors and anybody else who put money into the company and expects a return – is the following:

Free Cash Flow = EBITDA – Operating Taxes – Capital Expenditures – Working Capital Change + Deferred Tax Changes

In contrast to valuation approaches in corporate models that depend to a large extent on terminal value and cost of capital assumptions, project finance analysis in essence backs into the value of the prospective equity investment through assessing whether an adequate equity return can be achieved. The equity return in these models depends in turn on the amount of debt that can be lent to the project. This means that project finance models do not have the same problem that dramatically different results come from changing variables that are very difficult to estimate. Leveraged buyout models are similar to project finance models in the sense that one can back into the value of a company accounting for the amount of debt that lenders will commit to the transaction. With acquisition models value is computed through measuring the maximum entry multiple or premium that can be paid for a company such that a given the equity internal rate of return can be achieved. As with the project finance model, the rate of return to equity investors depends to a great extent on how financing of the transaction is structured, but not on terminal growth and subjective cost of capital estimates. Finally, merger integration models also back into the value of the target company and the acquisition premium. This time however, the value is derived through comparing earnings per share and other financial ratios before and after a merger. Value can be derived by determining whether earnings per share for the combined company increase whether

financial ratios of the new combined company remain strong enough to support the desired credit rating. Use of an integrated model to back into the value of a company is illustrated below. In this artificial example, the transaction results in increased earnings (accretion) and it also maintains the bond rating within the standards for a BBB company given the assumed business category.[9] If the equity consideration is increased or the amount of debt used in the transaction changes or the accounting treatment changes, the accretion may no longer exist.



The alternative approaches to classic free cash flow valuation from project finance models, leveraged acquisition models and integrated merger models eliminate the need for debating the weighted cost of capital or the terminal growth rates. Instead, market information on the structure of the financing and direct information on the required returns for equity holders forms the foundation of the valuation process. If value is to be computed using the alternative models, the financial models must address detailed aspects of how free cash flows accrue to debt and equity investors. The cash flow process is different depending on the structure of the transaction implying that alternative starting points, different time period conventions and distinctive calculations of the manner in which cash flow is dispersed to debt and equity investors must be incorporated in the alternative types of models.

The difference in valuation techniques that are implied from using the alternative models is summarized in the table below. Subsequent chapters will further discuss the advantages and disadvantages of using alternative valuations.

[9] The bond rating band is computed from tables provided by Standard and Poor's which show provide the range in financial ratios that are used to gauge bond ratings given a certain business risk (chapter 4).

Valuation Analysis in Alterative types of Models				
	Corporate Model	**Project Finance**	**LBO Model**	**M&A Integration Model**
Valuation from Model	Present Value of Discounted Free Cash Flow or Multiples	Investment Decision and Implied Value depends on Equity IRR versus Market Hurdle Rate	Entry Multiple and Acquisition Premium Depends on Equity IRR and Hurdle Rate	Acquisition Premium Depends on Earnings per Share Acretion and Debt Ratios
Key Valuation Parameters	Weighted Average Cost of Capital, Multiples, Terminal Growth	Debt Capacity, Debt Terms	Senior and Subordianted Debt Financing and Exit Multiple	Sources of Funds Used for Tranasction and Assessment of Credit Quality
Traditional Risk Assessment from Equity Perspective	Sensitivity Analysis and Scenario Analysis of DCF and Multiple Value	Sensitivity Analysis and Scenario Analysis of Equity IRR	Sensitivity Analysis and Scenario Analysis of Equity IRR	Sensitivity Analysis and Scenario Analysis of EPS Accrection and Credit Quality
Traditional Risk Assessment from Debt Perspective	Break-even Analysis to Determine Ability to Re-finance and Maintain Credit Rating	Break-even Analysis to Determine at what Point Cash Flow Cannot Service Debt	Break-even Analysis to Determine IRR on Senior and Subordinated Debt	Break-even Analysis to Determine Prospective Credit Rating
Monte Carlo Analysis with Model	Probability Distribution of EPS and DCF Valuation	Probability Distribution of Equity IRR and Probability of DSCR below 1.0	Probability Distribution of Equity IRR, Senior IRR and Junior IRR	

Computation of Free Cash Flow from Financial Statements

In all types of models, free cash flow is generally computed to establish asset value, although equity cash flow is the emphasis of project finance and acquisition models. Traditional finance theory suggests that valuation of an investment should be measured from the present value of cash flows as if the plant was financed without any debt -- that is, discounted free cash flow. After tax cash is what has value to investors; cash is what lenders need in order to be repaid; cash is what customers pay and employees want. The notion of using cash flow and a discount rate that are independent of financing is intuitive. Say you are valuing a manufacturing facility. If you take a tour of the factory, its value should be a function of its efficiency, its ability to produce good products, its construction cost, its cost to operate and maintain and other physical factors. Its value should not depend on the manner in which money was raised to pay for construction.

Without taxes, it is straightforward to demonstrate that free cash flow is equal to the cash flow accruing to equity holders plus the cash flow accruing to debt holders, minority owners, preferred shareholders and any other investors in a company. While it is well known to professionals familiar with valuation that enterprise value is equal to the present value of free cash flows discounted at the weighted average cost of capital, it is useful to review basic ideas underlying the notion. The net cost and benefits of alternative decisions are made by computing the present value of discounted free cash flow at the after-tax cost of capital which depends on the risk of the project. If the free cash flows (that accrue to all claimants, including debt, equity, preferred stock, minority interest and others) are discounted at a weighted average of the discount rates for each element, then the present value of free cash flows equal the sum of the present values of each claimant. The sum of the value of debt and equity and other claimants is defined as the enterprise value of a firm. This means that the present value of free cash flow at the weighted average cost of capital is equal to the present value of equity cash flows discounted at the cost of equity, plus the present value of debt discounted at the discount rate for debt and so forth. Equity value to

investors can be established from enterprise value by subtracting current market value of debt from the asset value.

Internal Rate of Return and Net Present Value

Decisions associated with investments should be gauged by considering whether marginal benefits of the investment exceed marginal opportunity costs. Marginal benefits of making an investment are the free cash flows that are generated when the plant operates. Marginal opportunity costs include the initial investment and the rate of return that could have been earned by investors if another investment with a comparable risk profile would have been made. This basic economic principle of assessing decisions with opportunity cost means that once after-tax cash flow is established, the value of an investment should be assessed by computing the net present value of the cash flows measured at an after-tax discount rate that reflects risk of the project. All else equal, with riskier the cash flows, the value of an asset is lower because the opportunity cost from making similar high risk investments is more. If we are investing in a risk investment and we were making another investment with comparable high risk, we would be able to achieve a higher return.

When presenting outputs in a financial model, risk and value of an investment can be assessed using with a risk-adjusted discount rate using two methods: net present value or internal rate of return. One way to assess the marginal benefits relative to the marginal costs of an investment is to compute the present value of free cash flows at a discount rate that reflects risks that would be present if only equity was used in financing the plant – i.e., it should not reflect the risks that are created by adding debt to a project. A second approach to compute the net economic value of a project is to compare the internal rate of return generated from the free cash flow with the cost of capital. Some finance texts distinguish between net present value and internal rate of return and suggest that a net present value criterion is superior to internal rate of return.[10] This is a distinction with little practical import in most real world financial analyses. Since the internal rate of return can be defined as the rate of return that causes the net present value of a project to be zero, comparison of the internal rate of return to the cost of capital is an effective and intuitive way to assess value. The internal rate of return tells managers what the level of the rate of return on their investment. Further, the internal rate of return is very easy to compute using spreadsheet programs and it can be determined for different time frames (for example, the IRR can be computed over a period of ten years rather than over the entire life of the project).[11]

Computing Internal Rate of Return (IRR) with IRR, XIRR and MIRR Functions

The fundamental definition of the internal rate of return is the discount rate that makes the net present value zero. A definition that is much easier to explain is simply that the internal rate of return measures the growth rate in investor cash. The growth rate in cash is after all the thing that investors should be most interested in. For example, if there is an cash outflow in the first period of 100 and a subsequent cash inflow of 120, then both the growth rate and the IRR are 20%, demonstrating that the IRR is the growth rate. Similarly, if the cash outflow is 100 and the cash inflow in second period is zero and the

[10] See Brealey and Meyers, Principles of Corporate Finance 4th Edition, McGraw Hill Inc., New York, 1991.

[11] The difference between project IRR – the internal rate of return earned on free cash flow -- and the all-equity cost of capital can be used to measure economic profit. Through multiplying the IRR difference by the cost of the project a measure of the economic profit earned by the investment can be obtained. Because electricity generating projects require a lot of up front capital, small differences between the earned return and the cost of capital can lead to significant economic profit. This is very similar to the traditional positive net present value rule whereby all projects with positive net present value should be selected. If economic profit is positive, the investment is generating positive value. Given that the components of free cash flow have been established in earlier chapters, most of the remaining discussion addresses how to compute the all-equity cost of capital for an electricity generating plant.

cash flow in the third period is 120, then the compound annual growth rate and the IRR are both around 9.54%. Again the IRR equals the growth rate. In measuring the growth rate in cash flow, an issue that complicates the calculation is the treatment of returns that can be earned on cash flow that occurs after the initial outflow and before the final outflow of cash flow. When equating the IRR with the growth rate, the IRR function assumes that any cash flow realized in middle periods can be re-invested at the IRR itself. For example, if you receive 100 in year 2, the IRR function assumes that this amount of cash can be re-invested and earn the same return as the computed IRR. This can be a very bad assumption, especially when the IRR is very high or very low. Instead, it may be much better to assume that the 100 may is re-invested at the overall cost of capital. The modified IRR computed with the MIRR function solves this problem. If the weighted cost of capital is used as the re-investment rate in the MIRR, and if the project earns its cost of capital, then the IRR is the same as the IRR. However, the variation in the IRR over different scenarios is less.

The modified IRR evaluates cash outflows and cash inflows in a different manner. The present value of the cash inflows are computed at a given financing rate. The future cash outflows are grown (or the future value is computed using a given re-investment rate. The modified IRR is then the implied compound growth rate for the present value of outflows relative to the future value of inflows as illustrated below:

MIRR = (FV of Inflows at Re-investment Rate/PV of Outflows at Finance Rate)^(1/periods)

When using the IRR and MIRR functions in excel, it is useful to realize that a few quirks can occur. If you are applying the IRR function, the number is not affected by whether there are leading or trailing zeros. However, when using the MIRR function, the answer is different (because the timing of the present value and the future value calculations.)

Verification of XIRR

Step 1: Convert the annual rate into a daily rate using the formula (1+Annual Rate)^(1/365)-1.

Step 2: Then compound using accumulated days, beginning with accumulated days of zero in first period (1+daily rate)^(accumulated days)

Step 3: Use the SUMPRODUCT command to compute the present value

Sharing Cash Flow and Hurdle Rates

Examples of management incentives using alternative approaches. Project finance uses revenue sharing in infrastructure transactions, flipping of the taxes in renewable energy transactions, management incentives in leveraged buyouts. Use a function because of complexity.

Terminal Valuation in Discounted Cash Flow Model

The discussion below addresses the gap between theory and practice in applying the discounted cash flow model. Real world problems addressed include determining stable relationships between depreciation and capital expenditures, treatment of deferred taxes and long-run estimates of changes in deferred taxes, consistency between working capital changes and growth rates, use of multiples in computing terminal values and other issues.

This appendix discusses issues that arise when computing terminal value in using a corporate model. The necessity of this assumption comes from the basic definition at unlike humans, corporations are assumed to last indefinitely. Also, unlike humans, corporations are assumed to reach an indefinite stable period in which everything is calm and growth rates in income stabilize. If companies do have stable

cash flow and earning, then the value of those prospective stable cash flows can be easily computed using a perpetuity formula. Frequently the terminal value is much more than the value of cash flows over the explicit period. This appendix demonstrates that subtle differences in computing the terminal value can have dramatic effects on valuation.

the terminal value is much easier to calculated and the valuation should be higher. If the valuation of a company depends on an aggressive terminal value assumption which assumes a high growth rate, the company obviously has more risk.

Analysts, consultants, investment bankers and others who perform discounted cash flow analysis using corporate models generally make a surprisingly pessimistic assumption that growth in cash flow once a terminal period occurs will be limited to the projected rate of inflation. The valuations depend on the assumption that companies will somehow “stabilize” to a tranquil zero real growth rate in a period of somewhere between five and ten years, perhaps after a smooth transition period until this supposed harmony is obtained. While the assumption is commonly made, it is difficult to come up with any company -- or person for that matter -- that has reached this kind of tranquil nirvana or has managed such a transition to equilibrium. Further, the valuation analysts do not seem to be concerned about the basic point that if all companies somehow reach this kind of equilibrium where there is no real growth in cash flow, no companies would contribute to real economic growth and the world economy would stagnate in a never-ending recession. An illustration of the type of growth rate assumptions made in classic DCF analyses is shown in the graph below.[12] Of course, the date at which the transition from short-term to long-term growth occurs and the length of the transition period is arbitrary.

The whole idea of assuming that companies reach equilibrium growth (often assumed to be zero in real terms) is questionable, much less imagining that one knows in what year the stable growth will occur and how long a fade period will last. While the general modeling of terminal growth rates is very subjective, at least when one structures a model with the base period, the explicit cash flow period and a fade period, the effect of changes in growth rates can be assessed. Given the difficulty in determining how long growth will last, the suggestion by some that the length of the explicit period does not matter is a bit absurd for most practical modeling problems.[13]

The assumption of zero real growth from the final forecast period can of course be modified and improved upon. As stated above, classic application of the DCF method in corporate models involves applying a stable growth rate -- generally the rate of inflation -- to the final year free cash flows. This implies the value of the company in the terminal period is expressed by the formula:

Terminal Value = Free Cash Flow in Terminal Period x (1+g)/(WACC –g)

The primary items of free cash flow – EBITDA, operating taxes, working capital and capital expenditures can all be handled in a more careful manner than this standard assumption. A summary of how assumptions can be derived for the period after the explicit cash flow period – period t+1 -- is listed in the table below:

[12] Growth rate in the transition period can be constructed from the equation: $g_t=g_{t-1}$ x[1/(short-term growth/long-term growth)]^(1/transition period).
[13] Find quote in McKinsey

FCF Component	Classic DCF	Problems	Improved Approach
EBITDA	Grow Final Year Cash Flow at Low Growth Rate	Growing the Final Year Ignores the Implicit Return on Investment in the EBITDA and the Relationship Between Cap Exp Growth and EBITDA Growth	Compute the Theoretical EV/EBITDA Driven by ROIC, Capital Expenditures/Depreciation, Tax Rate and Depreciation Rate
Working Capital	Grow at Final Year	If Revenue and Expense Growth Rates are Changing, the Growth Rate in Working Capital will not be Correct by Simply Applying the Terminal Growth to the Final Year Working Capital	Model and Additional Period After the Last Explicit Year, Where the Revenues and Expenses Grow at the Terminal Growth
Capital Expenditures	Either Use the Final Year Capital Expenditures or Set Capital Expenditures Equal to Depreciation	There is a Theoretical Relationship between Capital Expenditures and Depreciation that Depends on Growth and Depreciation Rates; this Growth Rate Does Not Equal 100% Unless there is No Growth	Compute the Equilibrium Capital Expenditures to Depreciation by Making a Simple Model that Extends for Many Periods. This Model is a Function of the Plant Life and the Growth in Capital Expenditures
Taxes	Apply EBIT x Tax and Ignore Deferred Taxes	Deferred Taxes Reach and Equilibirum Level that Should be Consistent with Cap Exp Growth	Compute Equilibrium Deferred Taxes as Function of Tax and Book Depreciation Rates and Income Tax Rate using a Similar Method as Above, with a Long-term Model that Reaches Equilibrium

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(1) To make a decision using internal rate of return, one should compare the earned return to a cost of capital percentage that reflects risks of the cash flows. In theory, the rate of return on free cash flow is assessed through measuring opportunity cost with a discount rate that is not affected by debt except for the tax effects. The discount rate that measures risk if no debt existed is known as the all-equity cost of capital. This is a similar concept to the weighted average cost of capital ("WACC"), but unlike the WACC which often changes when the debt leverage changes, the all equity cost of capital is entirely independent of from the leverage. Using the all-equity discount rate, the value of an asset does not depend on the financing of that asset because both the free cash flow and the discount rate are independent of the amount of debt in the capital structure.

(2) The cost of capital applied to future cash flows should account for contractual aspects of the investment that affect the future volatility in cash flow;

(3) The discount rate applied to free cash flow of a particular project is not the same as the discount rate applied to different investments made by the same company;

Rearranging, the formula becomes:

$$k = D_1/P_0 + g.$$

This equation implies that lower growth results in a lower estimated cost of equity capital as a lower discount rate must be used to arrive at the same share price. Because the model estimates the cost of capital from assumed investor valuation models, the inputs to the model -- dividend yield and the growth rate -- are not the direct drivers of risk and cost of capital as was the beta and the equity risk premium in the CAPM.

The model has many problems including the fact that long-term growth rates in dividends predicted by sell side analysts and often used in the model are generally optimistic; that earnings growth rates cannot be substituted for dividend growth rates when dividend payout ratios are changing; that assuming a company will stabilize after a certain time period presents subjective estimates as to when will

stabilization occur and what will be the long-term growth rate and whether the model should be applied using a sustainable growth rate.

Inference of cost of capital from an investor valuation model does not require use of dividend forecasts to establish stock price valuation. Other valuation formulas can be used to infer the cost of equity capital, instead of using the present value of projected dividends as the underlying valuation model.

Through re-arranging the P/E formula, one can compute the cost of equity from the P/E ratio. In this formula, the cost of equity capital is driven by the growth, the return on equity, and the forward earnings per share:

$$k = [EPS_1 \text{ x } (1\text{-}ROE/g)]/P_0 + g$$

If you want to apply either the P/E or the standard dividend discount model, you can use a file provided on the CD. After you enter ticker symbols, the file retrieves data from the Yahoo.Finance website and then computes the cost of equity using one of the two formulas discussed above. Growth rates come either from investment analyst projected five year growth rates which are often used in the analysis.